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Öğrenme Güçlüğüne Sahip ve İşitme Engelli Öğrencilerin Açı Kavrayışları

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ÖZET

Açı kavramı günlük yaşam için önemli ve işlevsel olan geometri bilgisi gelişiminin merkezinde yer alır. Bu bağlamda çalışmanın amacı öğrenme güçlüğüne sahip (ÖGS) ve işitme engelli (İE) ortaokul öğrencilerinin açı kavramına dair kavrayışlarını incelemektir. Bir durum çalışması olan araştırmanın katılımcıları amaçlı örnekleme yöntemlerinden ölçüt örnekleme ve kolay ulaşılabilir örnekleme yöntemleri ile belirlenmiştir. Araştırmanın katılımcıları biri 6. sınıf öğrenme güçlüğüne sahip ve diğeri 6. sınıf işitme engelli iki öğrencidir. Öğrencilerle yapılan bireysel klinik görüşmeler sonucu toplanan veriler çapraz durum analizi yöntemi ile analiz edilmiştir. Araştırmada elde edilen bulgulara göre, her iki öğrencinin de açı kavrayışının açının şekli ile sınırlı kaldığı, yine her iki öğrencinin de açıda yön kavramının farkında olduğu ancak ÖGS öğrencinin açı kavramını daha çok statik olarak algıladığı, açı ölçümü ve açı ölçü birimine dair yeterli bilgiye sahip olmadığı, İE öğrencinin ise açı ve açı ölçüsü kavramlarını ayırt edemediği ancak ölçülerine göre açı çeşitlerinin farkında olduğu söylenebilir.

Anahtar Sözcükler: Öğrenme güçlüğü, işitme engelli öğrenciler, açı kavramı, matematik eğitimi

Angle Conceptions of Students with Learning Disabilities and Hearing Impairments

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ABSTRACT

The angle concept is central to the development of geometry knowledge, which is important and functional for everyday life. In this context, the aim of this study was to examine the understanding of the angle concept of middle school students with learning disabilities (LDs) and hearing impairment (HI). The participants of this study, which was a case study, were identified with the criterion and convenience sampling methods. The participants were two students, one of whom with LDs and the other had a HI, each of whom was in grade 6. Data collected through individual clinical interviews with the students were analyzed using the cross-case analysis method. According to the findings of the study, the angle conception of both students was limited to the shape of the angle, and both students were aware of the concept of direction in the angle, but the student with the LD perceived the angle more statically, and did not have enough knowledge about the angle measurement or its unit. It was seen that the student with the HI confused the angle and angle measure, but was aware of the various angles according to their measures.

Keywords: Learning disabilities, hearing impairment, angle concept, mathematics education

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1. INTRODUCTION

Angle conception is important for students. It is a process that starts with representing the space topologically and continues with understanding Euclidean geometry (Devichi & Menuer, 2013). In Turkey, teaching about angle conception, which is at the center of geometry knowledge development, begins in grade 3 of the mathematics curriculum designed by the Turkish Ministry of National Education (MONE, 2018) and ends in grade 7 and 8 with an examination of the angle properties of polygons. In this context, angles appear in each grade starting from the grade 3, as one of the fundamental concepts of geometry. Additionally, it

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is crucial for students to realize the shapes of objects that they see in their environment and to explain the relation between the shapes and functions of the objects. This is closely related to the knowledge of the students about the angle concept. This can be exemplified by understanding the work of a pair of scissors, the reason why the Tower of Pisa looks that way, or the reason why the shade length changes at certain times of the day on a sunny day. One of the goals of teaching mathematics is to facilitate the lives of the students by being a good problem solver in their daily life and by improving their reasoning and association skills (MONE, 2018; NCTM, 2000). These special goals are valid for all students, and this especially becomes more important for students with learning disabilities (LDs) and hearing impairment (HI), who have difficulties in their daily life due to their disabilities. It is very important for them to develop all of these skills, as well as recognize and associate the objects they see in their environment with each other. Considering that LD students are inadequate in terms of their visual spatial abilities (Andersson, 2010; Grobecker & De Lisi, 2000; Mammarella et al., 2013) and HI students are qualified as visual learners (Dowaliby & Lang, 1999; Marschark & Hauser, 2012), it can be said that the skills that need to be developed and emphasized are similar for these students. For this reason, focusing on basic geometric concepts and measurements come into question. Thus, it becomes more important to investigate the knowledge of the students about the angle concept and their learning needs. Therefore, it was aimed in this study to examine the angle conception of students with LDs and HI.

1.1. Angle Concept

The angle concept is defined in the literature as the amount of rotation, as a portion of a plane (area) or a geometrical shape. For example, the angle concept is defined as:

- The figure formed by two rays with a common point of issue, the area between two rays (Keiser, 2004).
- The portion of a plane included between two rays with a common point of issue (Kieran, 1986).
- The amount of rotation until the end ray intersects with the issue ray (Marjanovic, 2007).
- [BA(U[BC) set, A, B, and C being three points on a plane (Argün et al., 2014).

It can be seen that these definitions are conceptually different from each other. In fact, an angle is the union set of the two rays with a common point of issue (Argün et al., 2014). According to this definition, the angle concept emphasizes not a measure as the rotation amount or an area covered in space, but the union set of the rays. It can be expected that students will have various misconceptions and conceptions about this concept, and even have trouble in understanding this concept, considering that there are many interpretations of the angle concept in the literature. Indeed, studies on the understandings that students have about the angle concept confirmed this (Clements & Battista, 1990; Clements & Burns, 2000; Mitchelmore & White, 2000; Govender & De Villiers, 2003; Keizer, 2004; Yazgan et al., 2009; Devichi & Munier, 2013). In the literature, it was stated that even students without any impairment have some problems, such as differentiating the angle and angle measurement from each other (Mitchelmore & White, 2000; Jones et al., 2002; Yazgan et al., 2009; Cunningham & Roberts, 2010), coordinating different aspects of the angle, such as static and dynamic (Kieran, 1986; Magina & Hoyles, 1997; Mitchelmore & White, 2000), and measuring and processing angles (Clements & Burns, 2000). However, in the literature, with regards to the understanding of students from preschool to secondary education (Keizer, 2004; Atebe & Schafer, 2008; Kaur, 2020), the studies generally pertain to the angle understanding of high achieving students (Bütüner & Filiz, 2017) or whether the angle understanding of the students varies by gender (Aydın, 2018). However, in the accessible literature, it was observed that there is a need for studies that provided some insight into the angle conception understanding of students with LDs and HI.

1.2. Students with Learning Disabilities and Hearing Impairment

A LD can be defined as demonstrating unexpected, untypical learning failure without certain reasons (Fusch et al., 2003). Helpful criteria to explain LDs comprise low achievement, lack of evidence for low achievement (such as education, environment, IQ, language) (Fletcher et al., 2007) and being unable to answer to standard education (Fusch et al., 2003; Fletcher et al., 2007). On the other hand, the Individuals with Disabilities Education Act (IDEA) defined LD as:

> *A disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, which disorder may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations* (IDEA, 2004, Sec. 300.8 (c) (10)).

Therefore, it can be said that individuals with a LD can have difficulty in distinguishing the sounds in verbal language, understanding the semantic structure, and using the language (Schoenbrodt et al., 1997; Gargiulo, 2003).

On the other hand, people with HI have serious problems in defining themselves due to the lack of feedback in the talking-listening process, in understanding a written text, and they constitute shorter and simpler sentences when compared to their hearing peers (Volterre & Erting, 1998). It is known that it is especially hard for these individuals to create an image regarding abstract concepts, and these conditions cause individuals with HI to have problems in many fields, such as learning, studying, having a profession, and forming social relationships (Lopez-Ludena et al., 2013). Students with HI (Wood et al., 1983; Traxler, 2000) fall behind their normally developing peers in academic performance and these students especially fail at learning mathematics, which is an abstract domain (Montague, 1995; Swanwick et al., 2005). The IQs of many students with HI are at the same level as the general public and they mostly do not have any cognitive problems (Turnbull et al., 2007). Moreover, students with HI can learn the same as their hearing peers, but they need more concrete experiences (Girgin, 2003; Karp & Howell, 2004). Students with HI have been identified as visual students in the literature (Dowaliby & Lang, 1999; Marschark & Hauser, 2012).

Among the students in need of special education, students with LDs and those with HI are regarded as two student groups closest to each other in terms of the similarity of their learning problems. While both groups of students have normal and supernormal IQ levels, their success levels are low due to their problems, especially in academic skills, such as speech and language development, reading, and writing. In addition, students with LDs and HI acquire knowledge similar to their peers, but experience more difficulties due to auditory-perception problems. Namely, it is believed that the learning problems of these student groups are not due to their intelligence levels, but rather due to having problems with audio-visual perception. Therefore, this study aimed to understand these students in terms of their mathematical conceptions and it was thought that the study would provide insight into mathematics learning in inclusive classes, which contain many disability groups. There are students with different types of obstacles in an inclusive class in Turkey (MONE, 2000 Ministry of Education Special Education Regulation, 67-matter of Cp 7). Therefore, when teaching mathematics, it is expected that the teacher is able to meet the needs of all of the students in the classroom. Because of this, it becomes important for the teacher to be aware of the differences and resemblances of these students. In this context, this study can be useful for inclusive classroom teachers in responding to student needs by examining the understanding that LD and HI students have of angles.

It can be said that studies conducted with students with LD and HI are generally limited to numbers and arithmetic operations (Kelly & Mousley, 2001; Woodward & Montague, 2002; Kelly et al., 2003). However, the advantages of different learning domains should be employed for students to be able to understand the everyday objects and their functions. If students can understand their environments, it will help them in their daily lives and eventually, in their professional lives. They will encounter angle models in their daily lives such as doors, windows, and scissors. Therefore, in order to provide helpful knowledge for the daily and professional lives of students, and to answer their learning needs, it is necessary to unravel their conception of the angle concept. In this regard, the aim of this study was to examine the angle conceptions of two students, one with HI and the other with a LD. This study sought to answer the following questions:

1. What is the conception of the angle concept for the student with a LD?
2. What is the conception of the angle concept for the student with a HI?

In this context, it was believed that the current study would contribute to the literature in 2 ways. First of all, investigating the angle concept is a new issue for the literature regarding students with LDs and HI. It was believed that examining the mathematical knowledge, thinking, and reasoning of the students, within the framework of different concepts, would provide an insight into how to help them learn math. Therefore, it would be beneficial to become closely acquainted with the learning characteristics of students with LDs and HI. By considering these two groups of students together in the study, it was believed that it would aid in the teaching process by revealing the similarities and differences of the students for teachers who may encounter a variety of students that require special education in an inclusive class. On the other hand, students who have different types of disabilities and understandings can also contribute to the teaching of the angle concept by revealing the misconceptions and thoughts that have not been reported in the literature.

2. METHOD

In this research, it was aimed to examine the understanding of students with LDs and HI of the angle concept, in depth, without any intervention. In this context, this was a multi-case study with a qualitative research design (Stake, 1995; Yin, 2013). The study cases herein were, separately, a student with a LD and a student with HI, and the unit of analysis was the understanding that the students had of the angle concept. For this purpose, it was deemed appropriate to consider the multi-case study (Miles & Huberman, 1994; Vanwynsberghe & Khan, 2007) as a research design, which provides an in-depth examination of the similarities and differences of 2 different cases, and compare them while maintaining their uniqueness.

2.1. Participants

Criterion sampling and convenience sampling methods were used in order to determine the participants. The criteria that were taken into consideration in the determination of the students were that they had a report from the Counselling and Research Centers that they had a LD or HI, and that they were in grades 3–8, which is considered suitable for examining the understanding of the angle concept (MONE, 2018). In this context, the study was conducted with two volunteer students, Fatih and Burak (names are pseudonyms). Fatih was a student with a LD, while Burak was a student with HI. Each student was male and in grade 6. Fatih stated that he “loved mathematics and his mathematics teacher”. Fatih stated he regularly “did tests and revised the lessons” to be successful in mathematics. It was observed that Fatih had difficulties with arithmetical operations, using some mathematical terms, grasping abstract subjects, and differentiating the names of colors. Contrary to his expression, it was observed that he could be quickly bored and lose attention in math lessons. Burak was able to hear and speak through the device in his ear. He was a very successful student in mathematics lessons. His family was very interested in his education. Even though he had no report, he was thought of as gifted by his teacher. According to his teacher, Burak was particularly successful in geometry and weak in verbal subjects in mathematics. Both students stated that they learned the angle concept in their inclusive classes.

2.2. Data Collection

Data of the study were collected through semi-structured clinical interviews. The interviews that were conducted individually were recorded on video and audio and lasted for about 40 min. Individual interviews were held in environments that the students were used to and were not distracted. The interviews with Fatih were conducted in a classroom belonging to the special education center, while the interviews with Burak were conducted in a classroom in his school. Before the interview, the students were informed about the aim of the study and the method of data collection. The video and audio recordings were taken with their approval. Paper and pen were provided to the students in the interviews so that they could explain their thoughts by writing or drawing. Therefore, the video and audio recordings, and notes taken by the students and the researchers constituted the data of this study. A researcher that knew sign language and who was also a mathematician conducted the interviews with the student with the HI and the video camera was placed during this interview to easily record the hand-arm gestures of the student.

2.2.1. Data collection tools

Questions were asked in clinical interviews about the perception, identification, and exemplification of the angle concept of the students with a LD and HI. Semi-structured clinical interview questions were prepared by the researchers, utilizing the literature. The pilot interviews were conducted with two students, one of whom had a LD and the other who had a HI, who were determined, according to same sampling method, as participants, to reveal whether the interview questions served the aim of the study and examine the comprehensibility of the questions by the students. As a result of the pilot interviews, the questions were reviewed and changes were made to some of the questions. For example, in the pilot interviews, the students were asked to form an angle by presenting line figures, but this question was found to be abstract and incomprehensible, especially for the student with the LD. Therefore, it was decided to ask the same question with concrete materials, that is, to ask the students to form an angle by giving them two items, pencils.

In the interviews, first, the questions about basic angle knowledge, such as the angle concept, opposite angle, right angle, corner and arms of an angle, and angle measurement were asked. For example, these questions were "What do you think about "angle"? If you were to give an example of an angle from this class, what would this example be? What does "The measure of an angle is 20° mean?" Additionally, some of the questions were asked to reveal possible misconceptions. For example, since it was stated in the literature that students have difficulties in coordinating angles statically and dynamically (Kieran, 1986; Magina & Hoyles, 1997; Mitchelmore & White, 2000), in order to reveal the insights that the students had about angles, the opening a door or turning the cover of a book was observed, and attention was drawn to the corners of a board in the class. Thus, the students were intended to interpret the angle concept. As another example, the students were asked to show the angles on the visual form in Figure 7 or to draw an angle with a measure of 0°.

2.3. Data Analysis

Data were analyzed with case-oriented analysis among the strategies of cross-case analysis (Miles & Huberman, 1994). In this research, the understanding of students with a LD and a HI of the angle concept were investigated through patterns and the relationships between these patterns obtained by examining the verbal or gesture-based answers, and the drawings of the students in detail. Each video and audio recording from the interviews were transcribed verbatim. Then, the transcripts and field notes from each student were analyzed separately for explicating patterns. Since there were a lot of gestures, which were the hand and arm movements in the flow of the conversation (McNeill, 1992), in the data collected from the student with HI, the analysis of these data was first conducted by the researcher that knew sign language. The gestures of the students regarding the angle and its properties were also interpreted by including them in the findings. Agreement correlation between the individual case analysis for each student was provided by comparing them. The agreement was reached after certain discussions in the analyses where an agreement could not be reached. Then, the angle conceptions of both students were compared by cross-case comparison. In brief, each case was considered as a whole and content analysis was carried out by first considering the effects, causes, and forms within a case (e.g., LD), and only afterwards was a comparative analysis performed between the cases (Ragin, 1987).

2.4. Validity and Reliability

In order to reveal the conception that the students had of the angle concept in a valid and reliable manner, an interactive interview environment was provided to enable the students to express themselves clearly. First, the students were met and conversed with before the interview, and then the purpose of the research, details of the interview, and expectations from the students were explained to them. After the students gave approval, the interviews began. The fact that the researcher who had an interview with the HI student knew sign language enabled effective communication with this student. The participants attend different schools and did not know each other. In this context, the possibility of the participants affecting each other was eliminated.

Expert opinion was acquired for the reliability of the data analysis and peer review was applied as an external control mechanism (Lincoln & Guba, 1985). The correlation coefficient between each researcher and mathematics educator was calculated, respectively, as 0.94 and 0.93. For example, the angle measurement seen in Figure 5 by Fatih was reinterpreted as a result of expert opinion. It was decided that since Fatih confused the angle measurement with the length measurement, he tended to measure the angle with his finger.

3. FINDINGS AND DISCUSSION

In this section, the identified angle understandings of students with LD and HI were presented and compared by discussing the similarities and differences among the students. Participant students referred the definition of angle, components of angle, measurability, opposite angle, equal angle, 0° angle, angle types, naming angle, and regions of angle regarding the angle concept. Table 1 summarizes the angle understandings of the students with the identified categories and subcategories.

Table 1.
Angle Understanding of the Students

Angle understanding	Categories	Subcategories	Fatih (LD)	Burak (HI)
Definition of angle	Inner region of polygons (rectangular region, triangular region, etc.)		x	
	Formal configuration	Wide, narrow, etc.	x	
		Relating the arc length to the angle	x	x
	Interpretation of the measure as an angle		x	x
	The space and opening are interpreted as angle		x	x
	Corner and intersecting lines		x	x
	Interpretation of the angular region (bounding the drawn arc)		x	x
	Intersecting two rays			x
	Interpretation as statically		x	
	Interpretation as rotation			x
Components of angle	Corner	Unaware being intersection point of two rays but having intuitively aware	x	
		Interpretation of the corner as a corner	x	
		Interpretation of the corner as a main point		x
	Arrow	Aware of the angle direction	x	x
		Shows direction of angle		
	Arm	Arm is a length	x	
		Arm is a ray		x
Measurability	Measuring with finger and ruler		x	
	Measuring with a compass			x
	Unable to remember the unit of measure			x
Types of angle	Equal angle	Aware of equality of the angles, but unaware of similarity of their directions	x	x
	Opposite angle	Awareness of direction	x	
		Round angle (360°)		x
	0° angle	The smallness (drawing parallel line segments)	x	
		The absence (drawing overlapping lines)		x
	270° angle	Unaware	x	x
	The other angle types (right angle, round angle, obtuse angle, straight angle, etc.)	Unaware	x	
		Aware		x
Naming the angle	Unable to read the angle		x	
	Reading the angle (corner point is in the middle)			x
Regions of the angle	Unaware		x	
	Aware			x

As seen in Table 1, both of the students defined the angle generally as the inner region of polygons, measure as an angle, the space and opening, angular region (bounding the drawn arc), and intersecting two rays. These definitions showed that the students had difficulty in explaining the angle. Both students perceive the angle as the region limited by arcs and arms of the angle and thought that a 270° angle did not exist. Both stated corner, arrow, and arm as components of angle. They measured the angle with a finger, ruler, or compass. However, it can be said that while the student with LD did not have sufficient knowledge about the types of angle, naming angles, or regions of angle, the student with HI did. In the sections below, the findings of the study were elaborated and explained.

3.1. Angle Conception of Fatih, the Student with the Learning Disability

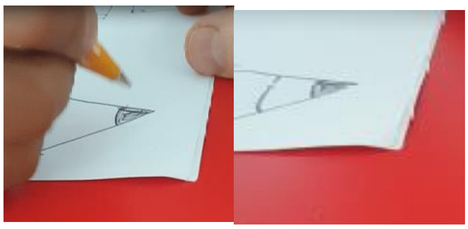
3.1.1. Definition of the angle

First, the definition of angle and what Fatih understood from this word was asked. Fatih answered this question as the "Angle of the square". When he was asked to draw it, he drew a figure resembling a rectangle that can be seen in Figure 1a and said that was the angle. Then, he shaded the corners of this figure (Figure 1a).

(a) (b) (c)

Figure 1. (a–c) Angle demonstration by Fatih

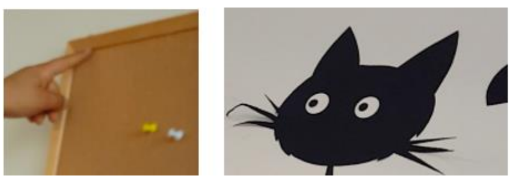
When he was asked "Is it an angle?" he pointed to the rectangular region and shaded the area with a pencil, saying "No, it's inside, inside". When he was asked to draw a triangle to show the angle, Fatih again shaded the triangular region for angle. As can be seen from the image, Fatih showed the angle by pointing the surface. Then, he was asked to show the angle on two intersecting rays. It can be seen in Figure 1b that Fatih drew an angle with his finger and said "Hmm exactly". He remembered angle from the drawn visual (drawn by the researcher) and showed angles on both the triangle and the rectangle (Figure 1c). Therefore, it can be said that Fatih could not remember the angle concept, but he remembered that it was a concept related to geometry; thus, he was inclined to draw geometrical shapes. The visual of two intersecting rays reminded him of the angle concept. When he was asked to show the angle on rays that intersected differently, he showed it by coloring (space and opening were interpreted as angle), as can be seen in Figure 2a.



(a) (b)

Figure 2. (a) Angle demonstration by Fatih and (b) Fatih questioning the angle

When Fatih was asked after drawing another arc whether these two were the same angle, he said that these two arcs defined different angles and when he was asked why, he said "This is narrow and other one is large". Therefore, it can be said that Fatih defined the angle concept formally, as in the literature, because Fatih only focused on a figural view of the angle while saying "narrow and large". Thus, he said that the arcs defined "the degree of the angle" as a finding that supported his conception. This situation showed that the student understood the arcs and region limited by the arms of the angle as the angle. When he was asked the meaning of the rays, in other words, arms of the angle; he first talked about the difficulty of the question and then said it meant "Length". In response, he was asked to show the angle again and he showed the space in between.
When Fatih was asked to give examples from the classroom that he was in, he showed right angle examples, as can be seen in Figure 3, as the angles of the bulletin board.



(a) (b)

Figure 3. (a, b). Angle examples by Fatih from the classroom

When he was asked to give different examples, he pointed the ears of the cat (Figure 3b). Therefore, it can be said that the main property that Fatih looked for in an angle was a corner. When he was asked where he pointed, he said "That corner". In addition, the intersection of two rays was drawn, and he was asked how many angles there were, to which he said that there was only one angle:

Researcher (R): Do you think there can be more angles here?
Fatih (F): No.
R: Why?
F: If it was like this triangle [shows the triangle], there would be three angles. But here [the rays], there is not something like this [draws an imaginary ray that intersects with other rays], a line.

Therefore, it could be thought that Fatih was aware that one of the key elements of angle is two intersecting rays.
When Fatih was asked to give examples of an angle using paper, he drew the arms of an angle and showed angles occurring at the corners, as can be seen in Figure 4.

(a) (b) (c) (d)

Figure 4. (a–d). Angle examples by Fatih

When the movement and question were repeated, Fatih looked for the angle at the right place, but did not show the opening between the two pages of the paper; instead, he showed the angle at the corner of the paper (Figure 4c). When the whole process was repeated, he showed the right angle, as can be seen in Figure 4d. However, what he meant to explain was the opening between the two pages. The student perceived the angle statically rather than the rotation amount, even though he realized that the angle formed with the rotation of paper after repeated questioning. When he was asked whether an angle was formed after folding the paper, he focused on static examples and could not completely explain the angle formed with rotation of the paper. When he was asked again about the angle formed with the rotation, he showed an angle at the corner of the paper.
R: For example, does this movement [rotation of a page] form an angle?

F: It does.
R: Can you show it?
F. …. Umm… I could not find it.

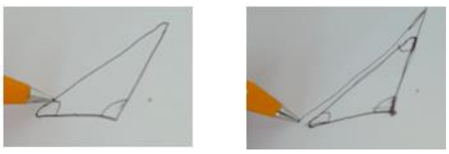
(a) (b)

Figure 5. (a, b) Angles formed by Fatih using rays

When he was asked to form an angle after two lines were drawn, he drew quadrilateral, as in Figure 5. In brief, it can be said that the student with a LD formed an angle with the angles of the quadrilateral and triangles and that was not aware of the main concepts and structure of the angle.

3.1.2. Components of the angle

A triangle-like figure was drawn for Fatih, as in Figure 6a. There was not a corner, that is, k-point, and when Fatih was asked whether the figure had an angle, he drew the angles by thinking of this figure as a triangle. From this, it can be said Fatih knew that the angle had corner.

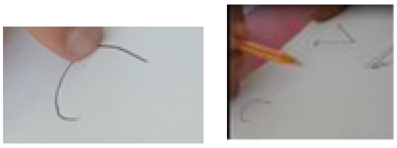


(a) (b)

Figure 6. (a, b) Angle demonstration by Fatih

Furthermore, when he was asked to show the corner of the angle in the drawings made before, Fatih showed them correctly. When he was asked to show the corners once more in the drawing in Figure 6a, he first showed the corners, and then showed the angles he drew. Therefore, it can be considered that he showed the other elements because he had insufficient knowledge on the concept of corner, or he could not find a corner according to the scheme in his mind. When previous examples were shown to Fatih, he decided that the corners in the last figure should be corrected and changed the figure by stating that it could not be like that (Figure 6b). When he was asked why, he said that "This one is crooked".

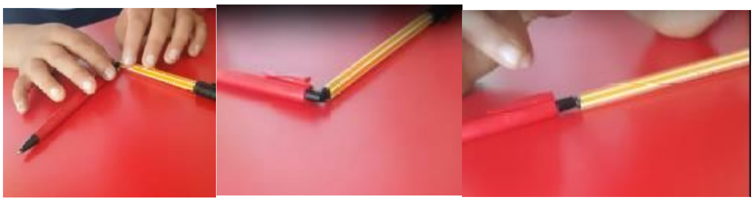
R: What happens if it is crooked?
F: It would be wrong.
R: For example, does this (Figure 7a) not form an angle?
F: It does.
R: Is this not crooked?
F: Right [he says and draws the figure in Figure 7b].



(a) (b)

Figure 7. (a, b) Fatih examining the situations that did not form an angle

Even though Fatih did not know that a corner was the intersection point of two rays, he was intuitively aware of it. When he was asked "Does the angle have anything to do with these (rays)?" He answered "No". However, he made an explanation for the arrows as "The arrows show the direction". Therefore, Fatih was aware of the direction concept in angles. He used the right parts in his angle drawings instead of rays. Two pens were given to Fatih, thinking that he could work better with tangible objects. Fatih formed the shape in Figure 8a.



(a) (b) (c)

Figure 8. (a–c). Angle examples by Fatih using pens

When he was asked to form an angle with these two pens, he drew imaginary arcs between the openings and said "I would form angles one after the other. I would form a lot". Based on this response, it can be considered that the angle conception that Fatih had was limited to arcs in introducing the angle conception or its symbolic demonstration. Apart from this, he changed the direction for different angles, as in Figures 8b and 8c. Therefore, Fatih was aware of the direction concept in angles.

3.1.3. Measurability

Fatih was asked about the measurability of an angle:

R: Can this angle be measured?
F: It can be measured.
R: What can we use to measure it?
F: Ruler. We can measure it by hand, but it would be a bit difficult.

When Fatih was asked to measure an angle, he tried to measure it with his fingers, as can be seen in Figure 9, but he did not continue, and said "I forgot".

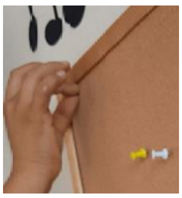


Figure 9. Angle measurement by Fatih.

It is not wrong for Fatih to try to measure the angle with his fingers. However, doing so by having a length measurement in mind can lead to the opinion that he answered without thinking. Furthermore, Fatih should have been careful to measure the right place. Fatih could not remember the angle measurement unit. It can be considered that Fatih did not have sufficient knowledge of angle measurement and angle measurement units. He read the symbol as “20°” but he said, as an explanation, that “For example, the temperature is 20° today”. Therefore, this knowledge could have been based on a Science lesson or everyday applications of air temperature. However, when he was asked by showing the 20° “Is this the angle itself?” He expressed his opinion as “It is not. Umm… it represents um… the degree of the angle”. Therefore, it can be said that Fatih was aware that the written statement was the result of the measurement. Furthermore, Fatih used his fingers to measure the angle and he used his finger as a unit, which can be expected from a student who has intuitional knowledge. It can be considered that Fatih confused the degree of the angle with unit of length, because he thought that it could be measured with his fingers, and thus, proposed to measure it with ruler. Eventually, he could not measure the angle with ruler. It can be said that Fatih did not have mental structures about the degree of an angle. This condition may have resulted from the lack of the mental structure on the scheme of the angle degrees and the difficulties that students with LDs face in using numerical data (American Psychiatric Association (APA), 2013).

3.1.4. Types of angles

Here, the understanding that Fatih had of the types of angles, such as opposite angle, equal angle, 0° angle and other types of angles (right angle, straight angle, etc.) were examined. For example, when Fatih was asked to give an example of the opposite angle, he showed the angle example seen in Figure 10a.

(a) (b) (c) (d)

Figure 10. (a–c) Angle examples by Fatih and (d) opposite and equal angles

For the opposite angle, he chose a different angle in terms of the direction. When he was asked to draw it, he drew the angle in Figure 10b. When he was asked to show the opposite angles on the figure drawn in Figure 10d, he said “All of them are opposite angles. One faces this direction and the other faces that direction”. Therefore, it can be considered that he was aware of the importance of the direction in opposite angles. Fatih was asked while drawing the angles in Figure 10d:

> *R: Would we able to say that all of these angles were equal if their degrees were the same, for example, 90°?*
> *F: Yes.*
> *R: [Question is repeated] Are they the same angles individually?*
> *F: [Thinks] No.*
> *R: Why?*
> *F: [By touching one of the angles] All of them need to be the same in order for them to be like this angle. One of these (arcs) big and other one is small.*

He was expected to realize that these angles were not equal, even if the angle degrees were the same due to having different directions. However, Fatih decided by focusing on the differences in the figure and according to his intuition, and he did not detect the equal angle correctly. Fatih was expected to realize that angles with the same degrees were not equal due to having different directions, but he decided by focusing on the differences in the shape and based on his intuition, even though it had been stated that the degrees were identical. Similarly, it was reported in the literature that students with LDs make mathematical decisions based on their perception, rather than tangible/numerical data (i.e. Güven Akdeniz, 2018; Mejias Mussolin, Rousselle, Grégoire, & Noël, 2012; Munro, 2003).

When he was asked to draw a 0° angle, he made the drawing in Figure 10c, and said “It must be so small”. It can be seen that Fatih drew two lines that were too close to each other. However, he could not intersect these lines. Therefore, his drawing was not consistent with his idea of looking for angle in the corner. However, it can be also thought that he drew two lines that were too close to each other because he understood it as the amount of space between the lines. That is, a 0° angle is small and between two parallel lines for him. When he was asked about obtuse, acute, right, equal, and straight angles, he said that he forgot and did not know. Fatih did not know the types of angles. When he was asked to draw angles with 270° and 360°, he said that “There is not an angle with 270°, there is only one with 360°”. When he was asked to draw that, he drew an acute angle (Figure 11). When he was asked why there is not an angle with 270°, he said that he did not know.

Figure 11. The 360° angle drawing by Fatih

When he was asked why he did not draw the arrows in this drawing and what the arrows meant, he said that "It indicates that the rays will continue" and completed the drawing. In the meantime, he was asked to make sure that he could use this idea in interpreting the angle:

R: Then, what does the arrow mean for the angle?
F: It's straightness.
R: Does the angle lose anything if the arrow is lacking?
F: It does not.
R: Why?
F: I do not know.
R: But there must be a reason to use the rays?
F: Angle cannot be found if it is crooked. There would be no angle.

His inability to evaluate the infinity of the rays for the angle was consistent with limiting angle to a certain arc. In addition, his emphasizing the straightness of the arms and the corner was important. However, he was not aware of the obtuse, acute, or right angles, and it can be said that his mental scheme to draw an angle according to the numerical degree was insufficient.

3.1.5. Naming the angle and regions of angle

The question of how many regions the angle divided the paper into, or reading or naming an angle with letters did not mean anything to Fatih and he said he did not know. The reason may have been that Fatih could not associate their meanings with symbols. Accordingly, it was reported in the literature that students with LDs could not associate number symbols with their numerical meanings-number sizes (Mejias et al., 2012). Additionally, this skill requires sorting the pieces of information, and it was stated that students with LD have difficulty in this regard (Munro, 2003).

3.2. Angle Conception of Burak, the Student with the Hearing Impairment

3.2.1. Definition of the angle

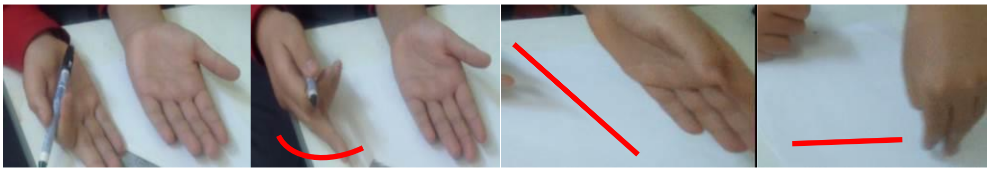
When Burak was asked what he understood from the word angle, he said that "I cannot define it, it is a mathematics subject" and drew the figure in Figure 12 and said that it was an angle.

Figure 12. Angle drawing by Burak

Although Burak could not verbally explain the angle, it was seen from his drawing and his explanations about the drawing that he drew the angle as the intersection of two rays. Moreover, he showed the figure of the angle and said "I will draw something that resembles the letter A, something like this" and "It will be in the shape of a triangle. It becomes an imaginary triangle when we close it here with our hand", after intersecting the end points of the rays with a line segment (Figure 13). Although it was hard for Burak to define the angle, the figures that he drew and the concepts that he used to define the angle drew attention.

Figure 13. The figure that Burak referred to as an angle

When Burak was asked whether opening the pages of the book formed an angle, he said, by thinking of the static examples and the angle formed with the rotation of the page that, "It is formed", and showed how it was formed by demonstrating "When we open the book like this [makes the gesture in Figure 14a], it is formed here [makes the gesture in Figure 14b: draws an arc with his right hand]. Then, the middle of it [makes the gesture in Figure 14c: draws a straight line with his left hand] is 180°, and the top of the page becomes [Figure 14d: draws a straight line from right to left with his left hand] an obtuse angle or straight angle according to the layout of the book" and explained his gestures. Burak mentioned that the rotation of the book pages formed an acute angle, while statically forming the straight angle with the intersection and the top of the pages. He showed the straight angle with a straight-line gesture, while showing the acute angle with an arc gesture. The gesture by Burak in Figure 14b, the arc drawn by the index finger of his right hand, meant that he showed the angle as "opening". Therefore, it was thought that Burak had "*gap, opening*" in his conception of angle. Burak showed the angle with his gesture in Figure 14b as opening, while showing the angle with his gesture in Figure 14b as the intersection of the rays. Thus, it can be considered from these gestures that Burak had confusion about the angle concept.


(a) (b) (c) (d)
Figure 14. (a) Gestures of the book pages, (b) drawing an arc, (c) a straight line, and (d) a straight line by Burak

When Burak was asked what the opening was and what it represented, he explained the distance (Figure 15a) between the arrows of the rays that form the angle as an opening. When he was asked if the angle and opening were the same things, he said that the opening (Figure 15b) was the angle. Even though Burak was aware that the angle and opening were used in different meanings, he was not aware that the opening between the rays of the same angle did not change in amount, while the openings that he described as the angle and opening represented the same amount. Burak made this explanation by focusing on the formal appearance of the figure.


(a) (b)
Figure 15. (a) The opening in the angle and (b) the angle according to Burak

Thus, when he was asked what the arcs meant by drawing the figure in Figure 16a, he said that the arcs were the interior angles. When asked whether their degrees were the same, he said that "Both are interior angles but their degrees are different. For example, while this place is 30° (Figure 16b), this place is 60° (Figure 16c)", and he also added that the degrees get higher. In other words, it can be said that Burak, like Fatih, perceived the region limited by the arcs and arms of the angle as an angle. However, it can be said that Burak thought the angle increased as the different arcs of the same span grew.


(a) (b) (c)
Figure 16. (a) The questioned arcs, and (b) 30° angle and (c) 60° angle according to Burak

In brief, it was observed that the student with HI could not define the angle, but he named it as two rays and one other main point, he was aware that the angle was formed with corners and his knowledge was sufficient in terms of the shape of the angle. Furthermore, it was stated that one of the biggest transformations for a student is mental image of the rotation (Clements & Sarama, 2014). Additionally, manipulating the objects visually is a part of the spatial skills (Zhang et al., 2012). However, the student with HI understood the angle by imagining the rotation mentally, while the student with the LD could not. This supported the result that students with LDs may have poor visual-spatial abilities in comparison with their peers (Mammarella et al., 2013).

3.2.2. Components of the angle

When Burak was asked to explain what the arrows and lines meant in his drawing, he said, by pointing at the corner of the drawing in Figure 12, "This is its main point; it is considered like that when we letter it". When he was asked whether the angle had a corner, he said that it did not have one. His explanation showed that the student did not know the corner of the angle, but could label this point as the "main point". Furthermore, he said, by pointing to the arms of the angle, that "This is the ray". Even though he knew the components of an angle, he repeated his statement before and said "I cannot verbally explain the angle". It can be seen that Burak had problems using the mathematical language. His description of the corner as the "main point" and the perpendicularity symbol in the right angle as "fitting the square at the corner" with verbal concepts were indicators that he had difficulties using the mathematical language correctly. Similarly, Girgin (2003) stated in their study that students with HI were affected by language and especially had difficulties using verbal language.

(a) (b)

Figure 17. (a) Straight angle and (b) acute angle by Burak

Burak was asked to form angles with two lines and he showed that he could draw a straight and acute angle, as can be seen in Figure 17. When he was asked the meaning of the arrow in the middle of the ray, which he drew in Figure 17b, he said that "That arrow does not have an end, it continues. I could make it continue like this". It can be said that Burak was aware of the direction concept in the angle according to this explanation. Furthermore, Burak could evaluate the infinity of the arms by pointing out that the arrow continued and he did not limit the angle with a certain arc. It can be considered that extending the arrow in a straight line referred to the linearity of the arms of the angle, and intersecting two lines referred to emphasizing the corner of the angle.

3.2.3. Measurability

Burak was asked about the measurability of the angle and he said that "It can be measured". Contrary to Fatih, Burak made a correct explanation by saying that the angle could be measured with a compass and he read the angle measurements as a "degree".

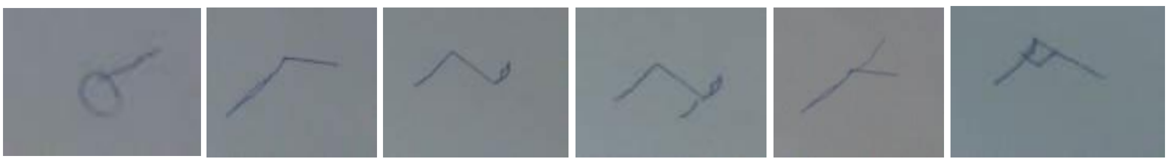
Figure 18. Measuring the angle with the compass

Burak said that the angle was measured with a compass. Then, he took a compass, (that is, the researcher did not give him a compass) and added "The compass has a small point (b-point) at its center. You can move it (Figure 18) however you want". Burak measured the angle with the compass by fixing one end (b-point) and moving the other arm (holding the a-point) and that was the correct application. Furthermore, Burak was asked what 20° meant and he said that "It means acute angle", "It is the angle itself". Thus, Burak was not aware that the written expression was the result of the measurement and that it was the quantity, that he confused the angle and measurement of the angle.

3.2.4. Types of angles

When Burak was asked to explain an opposite angle, he said that "An opposite angle is the same as 360°". When he was asked to give more details, he said that "If it is like a circle, it represents an opposite angle. So, this is an opposite angle" and then he drew the diagram in Figure 19a. Burak confused the opposite angle with a round angle. This may have resulted from his linguistic deficiency or from not paying enough attention to what was asked. When he was asked about the equal angles, he said that "It means equilateral. In other words, all angles are in the same degree". When he was asked for an example, he gave the example of an equilateral triangle. He showed the angles on the equilateral triangle and said that "This angle is 60°, this one and this one too. It means equilateral triangle. All of these interior angles are equal". Burak limited the equal angles to the equality of the measurement of the angles and he did not mention the similarity of their directions. Therefore, it can be considered that Burak identified the equal angle concept with equilateral triangle. Burak was expected to realize that angles with the same degrees were not equal due to having different directions, but he did not explain it as that. In this process, an obtuse angle was asked to the student:

> B: It means an angle over 90°.
> R: Can you draw it?
> B: This is an obtuse angle (Figure 19b), if we slide 90° like this (Figure 19c) it becomes an obtuse angle. If we slide it like this, it becomes an acute angle (Figure 19d). If we correct the obtuse angle like this (Figure 19e), it becomes a straight angle.



(a) (b) (c) (d) (e) (f)

Figure 19. (a) Opposite angle, (b) obtuse angle, (c) obtuse angle, (d) acute angle, (e) straight angle, and (f) right angle by Burak

Burak accepted the right and obtuse angles as the reference points while demonstrating the other types of angles and explained that the one over 90° was obtuse, the one below 90° was acute, and finally explained the straight angle by extending one of the rays of the obtuse angle. When he was asked about right angle:

B: I gave the example of the wall, it should be like this (Figure 19f), folded, in other words, the square must fit when placed. The folding should not overlap like this.
R: What does that mean?
B: 90°, it goes on like this in a straight line [shows the corners of the figure in Figure 19f], since it is straight, these straight lines must be equal with these straight lines [argues that the corners must be congruent].

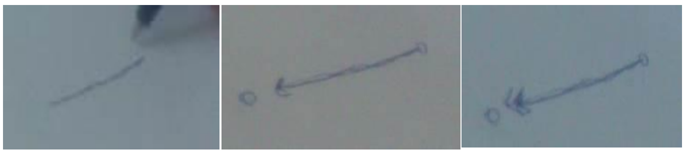
Burak explained the right angle as "The corners are straight and equal to each other" and could not explain that the corners must intersect each other perpendicularly by saying that "The square must fit when placed", and emphasized that the square must fit in the corner by associating the perpendicularity symbol with a square. Although Burak properly remembered the formal appearance of the right angle, he misinterpreted the symbolic representations of the figure. This condition showed that he focused more on the formal properties of the figures. Furthermore, his explanations showed that he was aware of the types of angles according to the degree. Furthermore, he said for the 270° angle, that "There is no 270° angle, only a 360° angle. And also, there are 180°, 90°, angles between 0° and 90° and angles between 90° and 180°. There are also acute angle, right angle, obtuse angle, full angle, and straight angles". When he was asked whether a 270° angle formed an obtuse angle, he stated that there was no angle between 180° and 360° and said "No. Obtuse angles are between 90° and 180°. There is also 360° of a full angle and that is it. There is no angle between 360° and 180°". Similarly, Fatih, the student with a LD, stated that there was no 270° angle and it was noteworthy that both students thought this way. The expressions of these students that the 270° angle and angles greater than 360° did not exist may have been due to in-class applications. The degrees of the angle were taken into account while naming the angles as acute, right, obtuse, straight, and round. When only these angles and degrees were included in the naming of angles, it could sometimes be difficult for students to realize angles other than these. For example, students may think that a 270° angle does not exist because of the fact that an angle label is not used for this angle. Therefore, Burak claimed that there were no angles with 0° or 270° as well.

When Burak was asked to give examples of the angle from the classroom, he said that "The corner of the door is the right angle". When asked why, he said, by making the gesture in Figure 20a, that "It completes 90°. It is straight and goes on like this".

(a) (b)
Figure 20. (a) Gestures of 90° angle and (b) straight angle by Burak

In this gesture, the elbow point of the left arm and the fingertips of the right hand are perpendicularly connected to each other. The area between the hands and elbows of each arm is thought of as a ray in this gesture. After that, Burak made the gesture in Figure 20b and said "Straight angle". When asked why that gesture represented the straight angle, he said "Because it is 180°". He drew a straight line in the air with the side of his hand, as in Figure 20b. Wilson (2002) claimed that gestures offered an objectified shape to convey a cognitive condition to the current environment. In other words, the gesture was the tangible indicator of the shapes and images that it represented. From this aspect, the gestures made for the right and straight angle reflected the 90° and 180° angles in the mind of the student. It was concluded from his statements that Burak knew that the angle was a measurable quality and that was aware that it was named differently according to its measurement.

Burak was aware of the acute, right, obtuse, straight, and round angles, he knew the degrees of these angles and he could draw any of these angles according to the degree. Even though Burak had difficulties in explaining the concepts, he did not have any problems with drawing the figures. Similarly, students with HI have been described as visual students in the literature (Dowaliby & Lang, 1999). Moreover, it was described in the literature that students with HI have difficulties in learning mathematical concepts in which verbal explanations are intense and complex (Nunes & Moreno, 2002), and they have better and faster development in geometrical concepts and skills than other learning domains (Pagliaro, 2015). They remember and improve spatial relations more easily (Zarfaty et al., 2004). The findings of the current study supported these results.



(a) (b) (c)
Figure 21. (a–c) The 0° angle drawn by Burak

When Burak was asked to draw a 0° angle, he drew the figure in Figure 21a and said "It does not exist, a 0° angle is like this". He also added "There are two arrows here (Figure 21b). One is like this, the other one is like that [showed the two overlapping arrows in Figure 21c]; there is no such thing as zero degrees". When his drawing is examined, it can be seen that he drew two overlapping rays as a 0° angle. Although his drawing was consistent with his search for the intersection of two rays in an angle, the expression "There is no such thing as 0°" made his statement contradictory.

It was thought that one of the questions in which the definition of angle can be questioned most effectively was the question when the student was asked to draw a 0° angle. It was also stated in the literature that students have difficulty with a 0° angle (Keiser, 2004). It can be said that the students had misconceptions consistent with the literature, such as the conception of a 0° angle or ignoring angles except for 90°, 180°, and 360°, but they also have different understanding. This was thought to stem from the perception difficulties of the students. Moreover, the different conceptions of the student with HI about equal angles, 0° angle, and right angle supported the opinion of Vygotsky, that a student with a certain disability is not a student that develops less than his/her peers, but is a student that develops differently (Vygotsky, 1993, p. 179).

3.2.5. Naming the angle and regions of the angle

Burak was asked to read the angle labeled with letters and he said that "B is the main point in $A\hat{B}C$ or $\hat{B}$, its combination is written in capital letters. It is shown as $A\hat{B}C$". When he was asked if this angle could be written as $C\hat{A}B$, he said that "It cannot be written. The main point is not letter A, it is letter B. The main point is at the end of the combination here. The main point is always written in the middle. It could be $C\hat{B}A$", and he made a correct explanation by indicating that the corner point must be written in the middle in the shape of Figure 22. Even though he used the statement of main point for the corner, he was also seen using corner for the main point. Therefore, it can be said from his explanation that he knew that the angle had a corner. In fact, he used the corner as the reference point in naming the angles.

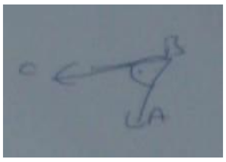
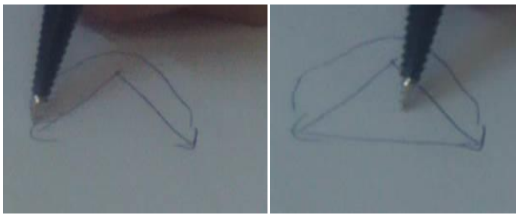


Figure 22. The figure used in naming the angle

For the question of how many regions the angle divides the paper into, Burak said that "It divides it into two zones. One, two" and showed the arms of the angle. In response, he was reminded of the inner and outer region concepts, Burak remembered these regions and he showed the regions on the angle (Figure 23). He said "Hmm I of heard, these are the outer region (Figure 23a), and these are exactly the inner region (Figure 23b). Interior angle and the inner region are the same".



(a) (b)

Figure 23. (a) Outer region and (b) inner region according to Burak

Accordingly, Burak was mistaken when the regions of the angle were asked about, and he showed the corners but telling him about the concept of the inner and outer region made him remember the regions of the angle. However, Burak said that by connecting the arrows of the rays that "The region inside the intersected arrows" and limited the inner region to the region between the rays. On the contrary, the arrows emphasized the infinity of the ray. This figure made by Burak suggested that he was not aware of the meaning of the arrows. When he was asked about the angular region concept, he said "I did not hear about it" and showed that he did not have knowledge on this subject.

In summary, the findings obtained about the understanding of the angle concept of students with LD and HI, and those summarized in Table 1, are as above. As a result of the semi-structured interviews with the students, findings on the definition of an angle, components of an angle, measurability, opposite angle, equal angle, 0° angle, types of angles, names of angles, and regions of angles were obtained, and the relevant categories were presented in detail. Considering the findings obtained in the study from a broad framework, when it was considered that students start learning about the angle as the main concept of geometry in grade 3 and they learn about naming, classifying, and drawing a given angle in grade 4 (MONE, 2018), it can be said that Fatih did not have sufficient knowledge as a grade 6 student, while Burak had sufficient knowledge when compared to Fatih. It is especially the aim of the curriculum to teach the student that the angle is formed by the rotation from the starting point of a ray using the compass (MONE, 2018). However, it can be said that Fatih perceived the angle too statically, while Burak associated the angle with the rotation by stating that it was measured with a compass. Even though the curriculum aims at this, it should not be forgotten that this situation was connected with the mathematics education received by Fatih in the classroom/learning environment. Furthermore, it cannot be ignored that Fatih is a student with a LD. It has been stated in the literature that students with LDs may demonstrate different development when compared to their peers (Vygotsky, 1993) and they can fall one or two years behind their peers in terms of expected academic achievements by standard education (APA, 2013). This was also observed for different mathematical concepts (Güven Akdeniz, 2018; Lewis, 2011). It is still considerable that the two participant students had same conception on certain points of the angle concept or that the student with HI had a different conception on some subjects. The reason for this may have been the difference in the reasoning skills of the students. Although they were influenced by verbal linguistic perception, the understanding that they had may have varied due to the tendency of students with LDs towards underspecification or overgeneralization (Güven Akdeniz & Argün, 2018), or as a result of their visual-spatial skills (understanding angle by imagining the rotation mentally). Indeed, it can be observed in the literature that students with LDs have weaker visual-spatial abilities when compared to their peers (Mammarella et al., 2013), while students with HI have better visual-spatial abilities (Rettenbach et al., 1999).

4. CONCLUSION AND IMPLICATIONS

In this study, it was revealed that students with LDs and HI had similar and different conceptions than their peers. In this context, it can be said that the misconceptions that may arise from the nature of the concept epistemologically may affect all students. Their different understandings are thought to be because of the verbal-auditory perception, attention deficit, memory problems, and overgeneralization-underspecification tendencies of students with LDs and HI. It can be said that the findings of the current study could expand the knowledge about misconceptions reported in the literature focused on the angle. Therefore, two of the main contributions of the study to the literature are providing insight into the nature and learning of the concept, as mentioned earlier, and the teaching of angles to be conducted in an inclusive class comprising students with LDs and HI. However, when comparing the angle conceptions of the two different disability groups, this study provides support for the teaching that should be conducted, by focusing on the differences of these students rather than the obstacles.

Although the confusion that the student with the LD had about the angle measurement and length is new to the literature, recognizing the quality in the measurement is one of the main characteristics. In this context, emphasis on the definition and exemplification of the angle is important for students to distinguish the angle measurement from the other qualities. The student with a LD expressing the opposite angle as direction for the opposite angle may indicate that he made inferences from the meaning of the word. He may experience difficulties in using geometric terms as well as being influenced by daily language, such as degrees or adjacent angles in the context of the angle concept. In this context, it is important that the mathematics teachers in the inclusive classrooms pay attention to the usage of mathematical language and examine what these mathematical terms mean to these students. Fundamentally, the student with the HI used the words as a result of being affected by the visual of the figure. It was seen that the student with the HI employed his gestures and figures drawn on the paper in explaining the concepts of the angle. The importance of sign language and gestures (Goldin-Meadow et al., 2012; Gürefe, 2015), in addition to visualization (Berndsen & Luckner, 2012; Ayantoye & Luckner, 2016) for students with HI has been mentioned in the literature. From this aspect, associating the concept with model and action (gesture) may make the message clearer, and more understandable for students with HI, and teachers are recommended to follow these characteristics of the students in classroom applications.

Additionally, the student with the LD could remember the angle with triangle and quadrilateral, while the student with the HI could talk about the properties of the figure by drawing it; the student with the LD made explanations based on his visual and intuitive perceptions, while the student with the HI made explanations based on his visual perception. When these are considered, it would be beneficial for students with LDs and HI to be introduced to the angle concept by giving examples from everyday life, such as the rotation of a compass or scissors, and employing concretization and visualization. Furthermore, Karp and Howell (2004) recommended that it is a useful approach to use unsuitable examples as well suitable ones in the education of individuals with special needs. From this aspect, it would be beneficial to use situations that form and do not form the angle, rather than giving one type of example in introducing the basic elements and properties of the angle. The fact remains that the results obtained were limited to the individual, environmental, and sociocultural factors of the participant students, and the cognitive tools (such as concepts, strategies, skills) that the students had (Sarama & Clements, 2009). In this context, the results of the study may be utilized considering student characteristics, thoughts, and the nature of the concept that was influenced by cultural experiences.

Research and Publication Ethics Statement

This is a research article, containing original data, and it has not been previously published or submitted to any other outlet for publication. The authors followed ethical principles and rules during the research process. In the study, informed consent was obtained from the volunteer participants and the privacy of the participants was protected.

Contribution Rates of Authors to the Article

Dilşad Güven Akdeniz: Data Collection, Data Analysis, Conceptualization, Methodology, Writing- original draft, Writing –reviewing & editing. Nejla Gürefe: Data Collection, Data Analysis, Conceptualization, Methodology, Writing- original draft, Writing –reviewing & editing. Ahmet Arıkan: Conceptualization, Methodology, Data Analysis, Writing –reviewing & editing.

Statement of Interest

There is no conflict of interest between the authors of this article.

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