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The Effects of a Functional Thinking Intervention on Fifth-Grade Students' Functional Thinking*

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ABSTRACT

This study aims to investigate the effects of a functional thinking intervention on 5th-grade students' functional thinking. Forty-three fifth-grade students from two public middle schools in Ankara participated in the study, in which 20 of them formed the experimental group in one school, and 23 of them formed the control group in the other school. The experimental group participated in a functional thinking intervention lasting 12 hours (about three weeks). A Functional Thinking Test (FTT) was administered to both groups as a pre- and post-test. The qualitative analysis of students' functional thinking strategies supported quantitative analysis. The results revealed no significant mean difference between the experimental and control groups at the pre-test or post-test. However, the experimental group showed significant pre-to-post gains. Also, experimental group students were significantly better at using variables in defining the function rule after the functional thinking intervention.

Keywords: Functional thinking, early algebra, recursive pattern, covariational thinking, correspondence thinking

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1. INTRODUCTION

The National Council of Teachers of Mathematics (NCTM, 2000) remarks on the importance of algebra competence in daily life and preparation for postsecondary education. It states that all students should learn algebra. The "arithmetic-then-algebra" approach, which is arithmetic in the elementary grades and algebra in later grades, has not been successfully helping students understand algebra (Knuth, Stephens, Blanton, & Gardiner, 2016). NCTM (2000) supported the idea of blending algebra in the curriculum from pre-kindergarten to help students construct a strong basis for understanding more sophisticated works in algebra in the future. The Common Core State Standards for Mathematics (National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010) signifies the role of algebraic thinking starting from kindergarten. Blanton and Kaput (2011) argued that students need early experiences to deepen mathematical structures and relationships rather than isolated computation exercises, and early algebra (i.e., algebraic thinking in early grades) provides these perspectives. Thus, students would become ready for algebra in later grades.

Blanton et al. (2018) described three core areas for early algebra: generalized arithmetic, equivalence, expressions, equations, and inequalities, and functional thinking. This study mainly focuses on the core area of functional thinking, which is defined as an essential way to algebra (Carraher & Schliemann, 2007). Functional thinking involves "generalizing relationships between co-varying quantities and representing, justifying, and reasoning with these generalizations through natural language, variable notation, drawings, tables, and graphs." (Blanton et al., 2018, p. 33).

Stephens et al. (2017) defined levels of sophistication for describing students' generalization and representation of functional relationships. The levels were categorized as three modes of functional thinking: recursive, covariational, and correspondence. Recursive thinking is when students define the recursive pattern by focusing on only one variable (e.g., the number of people

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goes up by 2). Covariational thinking describes the relationship between coordinated variables (e.g., while the number of tables goes up by 1, the number of people goes up by 2). Correspondence thinking involves writing function rules in words and variables (e.g., p=2 × t; If you multiply the number of the tables by two, you get the number of people who can sit at those). Stephens et al.'s (2017) levels of sophistication framework is also used in this study to analyze students' functional thinking strategies (see Table 4).

Many studies found that students can define functional relationships (also using covariational and correspondence thinking) and represent these relationships by using pictures, tables, graphs, words, and variables in early grades provided the appropriate environment and instruction (e.g., Blanton & Kaput, 2004; Blanton, Stephens, et al., 2015; Stephens et al., 2017; Türkmen & Tanışlı, 2019).

In Turkey, the algebra learning area is introduced in Grade 6, and the objectives related to functional thinking mostly take place in Grades 7 and 8 (e.g., Ministry of National Education [MoNE], 2018, M. 7.2.1.3, M.8.2.2.3). However, the focus on patterns in Grades 1-8 curriculum start in Grade 1 with a focus on geometric patterns (MoNE, 2018, M. 1.2.3.1, 1.2.3.2). Kabael and Tanışlı (2010), in their investigation of teaching patterns to functions, pointed out that teaching functions should be built on teaching patterns. While students learn patterns, it is also important to realize recursive, covariational, and correspondence relationships (İşler Baykal, 2019).

Students' understanding of the equal sign facilitates their understanding of expressions and equations in the middle grades and later (Blanton, Levi, Crites, & Dougherty, 2011); therefore, critical for functional thinking. Elementary students (Grades 3-5) were found to hold misconceptions regarding the meaning of the equal sign, such as seeing the equal sign as a signal for doing operations instead of signifying a relation (Stephens et al., 2013). Strachota et al. (2016) proposed that elementary students' functional thinking, especially representing functional relationships, may vary based on their understanding of the equal sign. Like the equal sign, students' understanding of variables is important for functional thinking (Blanton et al., 2011). Students may hold various misconceptions regarding variables (Küchemann, 1978). Therefore, in this study, the first week of the functional thinking intervention, which will be detailed in the method part and was developed based on the framework of Blanton et al. (2018), focused on the meaning of the equal sign and variables. This study aimed to investigate fifth-grade students' functional thinking and the effects of a functional thinking intervention on students' functional thinking.

1.1. Related Literature

Blanton and Kaput (2004) conducted a study to investigate elementary-grade students' expressions and representations from pre-kindergarten through fifth grade. "Eyes and Tails" task asked students, "If there was one dog, how many eyes would there be? Two dogs? Three dogs? 100 dogs? Do you see a relationship between the number of dogs and the number of eyes? How would you describe this relationship?" (p. 136). Pre-kindergarten students worked by counting visible objects (dog pictures, dots for eyes, and marks for tails) instead of using patterns and predictions. Some students described the pattern as "counting by twos" and "every time we add one more dog, we get two eyes." (p. 137). First-grade students defined multiplicative relationships as "double" (eyes) and "triple" (eyes and tails). Second graders recorded data on ten dogs on a t-chart and defined the multiplicative relationship using natural language. Also, they could find the number of eyes and the number of tails for 100 dogs. Third graders described the rule in words and variables as "*n×2*" and "*2×n*". One of the third-grade students drew a line graph to show the relationship. Fourth and fifth-grade students could define the functional relationship by using fewer data. Besides, a fourth-grade student wrote: "□× 3 = n" (p. 140) to represent the relationship between the number of dogs and the total number of eyes and tails. This study showed that students could think covariationally, even in kindergarten. Also, even third-grade students could define the relationship using symbols, variables, and words.

Blanton, Stephens, et al. (2015) implemented a comprehensive early algebra intervention for third-grade students through 19 lessons once a week. They found that experimental group students showed a significant gain, and there was a significant difference in post-test between experimental and control groups. Experimental group students outperformed control group students in defining a covariational relationship, function rule in words, and variables for the relationship between the number of tables and the number of people at Brady's Birthday task. More experimental group students defined the function rule in variables (e.g., A × 2 + 2 = B) than words (e.g., number of tables times two plus two equals the number of people (16% vs. 8%, respectively) at post-test in item d (p. 67). All in all, functional thinking intervention had positive effects on students' functional thinking even in third grade.

Stephens et al. (2017) focused on students' functional thinking and representations as a part of a three-year longitudinal study on early algebra. Students' responses were analyzed through a coding schema based on the levels of sophistication describing students' generalization and representation of functional relationships (see Table 4). Stephens et al. (2017) found that students succeeded in defining function rules in variables more than in words, so the categories of words are placed at a higher level. Moreover, while students' responses were at L2 (recursive pattern) in the beginning, those shifted toward L6 (Functional Basic), L9 (Functional Condensed in Variables), and L10 (Functional Condensed in Words) (see Table 4).

Pinto, Cañadas, and Moreno (2021) conducted a study with 24 third-grade students to investigate how third-graders define and represent functional relationships. Students mainly defined correspondence relation by natural language and numerical representation for *y=ax+b* functional relationship. Students' answers were analyzed according to categories of functional relationships (recursive patterns, correspondence, covariation) and representations (natural language, manipulative, pictorial, numerical, algebraic notation, tabular). General results showed that third graders could generalize relationships between variables for the functional relationship they had not faced. In addition, most of the students tended to define correspondence relationships by using specific values, and they had difficulty generalizing functional relationships. On the other hand, three students could generalize the functional relationship. Natural language and numerical expressions were the main representations used by students.

In Turkey, Tanışlı (2011) focused on fifth-grade students' functional thinking ways by linear function tables. Students were found to look down the tables and define recursive patterns. When they looked at the tables horizontally, they could realize the correspondence relationships and generalize this relationship using words and semi-symbolic forms. Türkmen and Tanışlı (2019) conducted a study to investigate 3rd, 4th, and 5th-grade students' levels of generalizations of functional thinking in the early grades. This study focused on two functional relationships: *y=2x* and *y=2x+2*. For the task *y=2x*, most students (46.7 % of 3rd graders, 44.4 % of 4th graders, 34.2% of 5th graders) were found in the "Functional Particular-Multiplicative Relationship" level. At this level, students could define the multiplicative relationship between the number of tables and the number of people as "1×2=2, 2×2=4, 3×2=6", but they could not generalize this functional relationship. Moreover, 31.4 % of fifth-grade students defined the function rule as "M×2=K". For the task *y=2x+2,* students tend to ignore the constant term. While the majority of students (28.9% of 3rd graders, 33.3% of 4th graders, 22.9% of 5th graders) defined wrong multiplicative relationship, about 18% of 3rd grade, 25 % of 4th grade, and 14% of 5th-grade students defined the correct rule in the "Functional Particular-Multiplicative Relationship" level by regarding the constant term. Therefore, Türkmen and Tanışlı (2019) suggested that functional thinking should be placed in the curriculum in earlier grades.

1.2. Purpose of the Study

All in all, related literature revealed that students can describe functional relationships and represent them in multiple ways, such as pictures, tables, graphs, words, and variables, in early grades. Their functional thinking can be developed through a learning environment that supports functional thinking. Although the Turkish Mathematics School Curriculum (MoNE, 2018) focuses on patterns, it is important that students realize recursive, covariational, and correspondence relationships via patterns. Moreover, in the Turkish literature, although there are studies that aimed to find out students' functional thinking capacities (e.g., Tanışlı, 2011; Türkmen & Tanışlı, 2019), there are limited studies that focus on developing students' functional thinking. In this regard, this study aimed to reveal the effects of a functional thinking intervention on fifth-grade students' functional thinking. Also, the study aimed to identify students' functional thinking strategies based on the framework developed by Stephens et al. (2017).

Four research questions were investigated in the present study below.

1. Is there a statistically significant mean difference between the functional thinking post-test scores of the 5th-grade students who attend the functional thinking intervention and those who do not attend the functional thinking intervention?
2. Is there a statistically significant mean difference between the functional thinking pre-test and post-test scores of the 5th-grade students who attend the functional-thinking intervention?
3. Is there a significant relationship between the two groups (5th-grade students who attend the functional-thinking intervention and those who do not attend the functional thinking intervention) and the correctness of the functional thinking test items at pre-test and post-test?
4. How do 5th-grade students' functional thinking strategies differ in the functional-thinking test for those who attend the functional-thinking intervention and those who do not attend the functional thinking intervention?

2. METHODOLOGY

2.1. Research Design

The current study focused on the effects of a functional thinking intervention on fifth-grade students' functional thinking. The static group pretest-posttest design was used with two intact groups (Fraenkel, Wallen & Hyun, 2012). The groups were not randomly assigned to the conditions and were chosen as two classrooms in two public schools. The experimental group attended the intervention, and the control group did not receive an intervention. Both groups received a Functional Thinking Test (FTT) as a pre-test, and then experimental group students attended a functional thinking intervention for about three weeks. Both groups took the same FTT as the post-test at the end of the intervention.

2.2. Participants

The participants were fifth-grade students from two public secondary schools in Çankaya, Ankara. Those schools were selected by convenience sampling method. Schools' physical conditions were similar. However, in the control group, students were enrolled in school according to their primary school GPAs, so students' academic level might be higher than the average. In the control group, there were 23 students: 11 girls and 12 boys. In the experimental group, there were 20 students: 7 girls and 13 boys. The experimental group was more diverse than the control group. There were a non-native student and an inclusive student in the experimental group. While the inclusive student did not participate in activities, the non-native student participated in the activities with the language support of the instructor.

2.3. Data Collection

Through the aim of the study, students participated in a pre-test and a post-test, which were identical. Students were allowed 40 minutes to take the tests. Experimental students participated in a functional thinking intervention for about three weeks (12 lesson hours) between tests. After getting approvals from the Human Subject Ethics Committee at the university, the Ministry of National Education (MoNE), and parents, data were collected in the Spring semester of the 2018-2019 academic year.

2.3.1. Instrument

The study aimed to investigate fifth-grade students' functional thinking and the effects of a functional thinking intervention on students' functional thinking. Through this aim, a Functional Thinking Test (FTT) (see Appendix A) was designed by the researchers based on the objectives related to mainly the functional thinking learning goals covered in Blanton et al. (2018) (see Table 1). The FTT had two main problems related to $y=mx$ and $y=mx+b$ types of equations. Through these questions, students were expected to identify data, organize the data in a table, define patterns in this table, define the rule of the relationship between two quantities in variables and words, and draw the coordinate graph to show the relationship and use function rule to find near and far data.

2.3.2. Intervention

To investigate the research questions, the researchers designed a functional thinking intervention. After implementing the pre-test in both groups, experimental group students participated in the intervention for 12 lesson hours (about three weeks). The control group was subjected to the regular curriculum focusing on geometry concepts. They did not receive a functional thinking intervention. There were 5 lesson plans based on the instructional sequence in Blanton et al. (2018, see Table 1). Blanton et al. (2018) constructed their instructional sequence based on three core areas: (1) generalized arithmetic; (2) equivalence, expressions, equations, and inequalities; and (3) functional thinking. The current study focused on the goals regarding the functional thinking part. Also, the meaning of the equal sign and the meaning of variables were handled in the first lesson due to their importance for functional thinking mentioned in the introduction. All lesson plans were developed using three instructional methods: questioning, discussion, and group work. These methods were implemented parallel to the intervention developed by Blanton et al. (2018). Students were usually asked to work in groups in the activities, followed by a whole-group discussion led by the researcher. Each lesson had an activity sheet and an exit card. Lesson plans were developed based on $y=x$, $y=2x$, $y=x+1$, *and* $y=2x+1$ functional relationships, respectively. Students were expected to define functional relationships between two quantities in different ways: using words, tables, variables, and graphs. The meaning of the equal sign, the meaning of unknown, and variables, and using them in an equation as prerequisite knowledge in the scope of the first lesson plan was handled in the first lesson. In the second lesson, functional thinking-based activities started, and the lesson sequence was the same for each of the following lessons. Students were asked to construct a table to show different values for given variables. Then, students worked on defining the patterns in the table and functional relationships between variables. Students were asked to define and describe the function rule in words and variables. Lastly, students were asked to use a function rule to predict near and far data. For some lessons, students were also asked to draw a coordinate graph to represent functional relationships. Next, a sample lesson, the fifth lesson (see the lesson plan in Appendix B), will be detailed below.

Table 1.
The Instructional Sequence of the Functional Thinking Intervention

Lesson	Goal of Lesson
1st lesson (2 lesson hours)	Examine the role of the equal sign; the relational meaning of the equal sign "Identify a variable to represent an unknown quantity." "Examine the role of variable as a varying quantity." "Represent a quantity as an algebraic expression using variables." "Interpret an algebraic expression in a context."
2nd lesson (2 lesson hours)	"Generate data and organize in the function table." "Identify variables and their roles." "Identify a recursive pattern, describe in words." "Identify covariational relationship and describe in words." "Identify function rule and describe in words and variables" (The type of function: $y=x$) Use a function rule to predict near and far data.
3rd lesson	"Generate data and organize in the function table." "Identify variables and their roles."

(3 lesson hours)	"Identify recursive pattern, describe in words." "Identify covariational relationship and describe in words." "Identify function rule and describe in words and variables" (The type of function: *y=2x* and *y=3x*) "Use a function rule to predict near and far data."
4th lesson **(2 lesson hours)**	"Construct a coordinate graph to represent problem data." "Generate data and organize in the function table." "Identify variables and their roles." "Identify recursive pattern, describe in words." "Identify covariational relationship and describe in words." "Identify function rule and describe in words and variables" (The type of function: *y=x+1*) "Use a function rule to predict near and far data."
5th lesson **(3 lesson hours)**	"Generate data and organize in the function table." "Identify variables and their roles." "Identify recursive pattern, describe in words." "Identify covariational relationship and describe in words." "Identify function rule and describe in words and variables" (The type of function: *y=2x+1* and *y=2x+2*) "Use a function rule to predict near and far data."

Note. Adapted from "Implementing a Framework for Early Algebra" by M. Blanton et al., C. Kieran (ed.) *Teaching and Learning Algebraic Thinking with 5- to 12-Year-Olds*, ICME-13 Monographs, pp. 36-37, 2018, Springer Cham.

The fifth lesson covered the *y=2x+1* functional relationship by the String Task (see Figure 1). The researcher made groups of four and distributed an activity sheet, a scissor, and four different colors of ribbons: red, yellow, blue, and pink to each group. There was a knot in the center of all ribbons.

Figure 1. The String Task (Taken from Isler et al., 2015, p. 285)

All groups and the researcher cut the red ribbon together. The ribbon was folded from the middle; then, the ribbon became double-deck and was cut once. The researcher asked students how many pieces there were. They got three pieces; one of those was a knotted piece. Students continued working on cutting with other ribbons. Then, students recorded the number of cuts and the number of pieces they got. Students could just define a recursive pattern as *"The number of pieces increases by 2."* They could not find any covariational and correspondence relationship in the table. Therefore, the researcher created a different table including columns: the number of cuts, the number of pieces without a knot, the number of knotted pieces, and one column for the total number of pieces as described in Isler et al. (2015). All groups wrote their findings in the table. Then, the researcher asked students about the relationship between the number of cuts and the number of pieces. One of the groups defined a covariational relationship as "*As the number of cuts increases by one, the number of pieces increases by two."* The other described a correspondence relationship as *"The number of the pieces equals two times the number of cuts."* In the previous lessons, students worked on defining the y=2x functional relationship so they could generalize the functional relationship as *"K×2=P"* and *"P=2×K."* "K" showed the number of cuts, and "P" showed the number of pieces. After all, the researcher asked, "*What about the knotted piece?*" Thus, students realized that there is one knotted piece in all cases. Through discussions and the researcher's help, students noticed that one knotted piece was added in each case; the relationship between the number of cuts and the number of total pieces was defined as "*P=2×K+1.*" All activities in the lessons were performed in the same manner. Firstly, students worked in small groups, then by whole class discussions, students' answers were assessed, and the functional relationships were defined by multiple representations using tables, words, and variables.

2.3.3. Data analysis

Data were analyzed by using both quantitative and qualitative analysis methods. Quantitative analysis was conducted via statistical tests at IBM SPSS 24. To respond to the first research question, the Independent Samples T-test was used. In this study, the sample size of both groups was smaller than 50, so the Shapiro-Wilk Test was conducted to check the normality assumption. For the second research question, the Dependent Sample T-test was conducted, and the normality assumption was checked by the Shapiro-Wilk Test. Also, the Chi-Square Test for Independence was performed to respond to the third research question. There were no assumptions except minimum expected cell frequency, which will be mentioned in the findings, for the

Chi-Square Test for Independence. For the fourth question, through the qualitative analysis, students' responses were coded based on correctness and their strategy use. A codebook was created based on Stephens et al.'s (2017) levels of sophistication in functional thinking (see Table 4). While students' correct responses were rated as 1, incorrect responses or no responses were rated as 0. There were 14 items in the FTT, so students' scores ranged from 0 to 14.

2.3.4. Validity and Reliability

There were some threats to internal validity in this study, such as mortality, instrumentation, testing, and implementation (Fraenkel et al., 2012). Although all experimental group students attended the pre-test and post-test, there were two losses of control group students. To minimize the testing threat, there were approximately three weeks between the pre-test and post-test. The instrumentation threat includes data collector characteristics and instrument decay. In the study, the same researcher collected pre-test and post-test data, implemented the intervention, and analyzed the pilot and main study data. Therefore, the data collector characteristics threat was eliminated. To prevent instrument decay, a codebook was created, and students' answers were coded following this book. The present study was conducted by a non-random sampling method so that the generalizability of the results to the population was limited.

Inter-coder agreement method was used to increase the coding reliability. Twenty percent of the data (four students from the experimental group and five students from the control group) were randomly selected, and students' responses were coded for both correctness and strategies by two researchers. At least an 80% agreement was reached for all items. Any discrepancies in coding were discussed and agreed on by the researchers.

3. FINDINGS

In this section, the Functional Thinking Test (FTT) results are presented in two parts: inferential statistics results and descriptive results. We present students' overall assessment, performance, and strategies they used for selected items. Inferential statistics results show whether there was a mean difference between the scores of the experimental and control groups. Independent-samples T-tests were used to investigate the mean difference between the groups at the pretest and posttest. Paired-samples T-tests were sought for the difference within the groups. Descriptive results present students' strategies.

3.1. Inferential Statistics Results

To respond to the first research question, having met all assumptions, an independent-samples T-test was conducted to compare the pre-test scores for experimental and control groups. The normality assumption was checked by the Shapiro-Wilk Test. There was no significant difference in scores for the experimental group (M= 4.75, SD = 1.99) and the control group (M= 5.65, SD = 1.89; t (41) = −1.52, p = .14, two-tailed). Regarding the group differences in the post-test scores, the homogeneity of variance assumption for the T-test was not satisfied; therefore, the second line of the T-test table was used (Pallant, 2005). There was no significant difference in scores for the experimental group (M = 7.05, SD = 3.53) and the control group (M = 6.17, SD = 2.12; t (30.25) = .97, p = .34, two-tailed) in the post-test.

In addition, to answer the second research question, having met all assumptions, a paired-samples T-test was used to investigate whether there was a significant mean difference between pre-test and post-test scores for the experimental group. The normality assumption was checked by the Shapiro-Wilk Test. Paired-samples T-test showed that there was a statistically significant increase in test scores from the pre-test (M = 4.75, SD = 2.00) to the post-test (M = 7.05, SD = 3.53), t (19) = 2.81, p < .05. The mean difference in test scores was 2.30, with a 95% confidence interval ranging from .59 to 4.01. The eta squared statistic (.29) indicated a large effect size.

Moreover, having met all assumptions, the paired samples T-test showed that there was no statistically significant increase in test scores from the pre-test (M = 5.68, SD = 1.94) to the post-test (M = 6.40, SD = 2.32), t (21) = 2.01, p > .05 for the control group. The mean difference in test scores was 0.72, with a 95% confidence interval ranging from -.02 to 1.48.

In addition, Chi-Square tests were used to answer the third research question. A Chi-square test for independence (with Yates Continuity Correction) indicated that there was a significant relationship between the experimental and control groups for Item 1e, $\chi 2$ (1, n = 43) = 5.6, p = .018, *phi* = - .415. In the case of violating the 'minimum expected cell frequency' assumption, Pallant (2010) suggests using Fisher's Exact Probability Test values so that the Chi-square test for independence (with Fisher's Exact Test) indicated that there was a significant relationship between the experimental and control groups for Item 2e in the post-test (.016 < .05). The experimental group significantly outperformed the control group using variables to define the function rule in both items.

In conclusion, statistical results showed that although there was no significant mean difference between the post-test results of experimental and control groups, there was a significant mean difference between the pre-test and post-test scores within the experimental group. Chi-square tests for independence showed that the experimental group significantly outperformed the control group using variables to define the function rule in items 1e and 2e, specifically using variables to define the function rule in both items.

3.2. Descriptive Results

Students' answers to the tests were assessed by correctness and strategy use. Qualitative analysis was performed to investigate whether students' functional thinking strategies differed in the functional-thinking test for those who attended the functional-thinking intervention and those who did not attend the functional-thinking intervention. Both Item 1 and Item 2 had seven sub-questions. For many items, a correctness (correct [1], incorrect [0]) code and a strategy code were assigned. Coding schemes varied according to the structure of the items. For items that asked to define patterns in the table, function rule in words and variables, levels of sophistication for generalizing functional relationships (Stephens et al., 2017) were used. For items asking to find near and far value using function rule and other strategies, emerging codes, and strategies from literature such as Blanton, Stephens, et al. (2015) were used. Apart from these, "Answer Only (AO)," "No Response (NR)," and "Other (O)" codes were utilized. The frequency of correctness of students' answers for each sub-item in the first main problem was presented in Table 2.

Table 2.
Correctness of Students' Answers for Item 1

	Experimental Group N=20		**Control Group N=23**	
Items	**PRE**	**POST**	**PRE**	**POST**
Item 1a	85%	90%	100%	95.65%
Item 1b	90%	95%	100%	100%
Item 1c	80%	85%	78.26%	86.96%
Item 1d	5%	30%	39.13%	34.78%
Item 1e	0%	45%	0%	8.70%
Item 1f	90%	95%	82.61%	95.65%
Item 1g	0%	0%	0 %	0%

Experimental and control group students showed similar performance for the first main problem. Most experimental and control group students found near values for the given variables correctly completed the table and defined patterns (items 1a, 1b, and 1c). However, while the correct answers in describing the function rule in words (item 1d) decreased at the post-test in the control group, it increased in the experimental group. Moreover, no students could define the functional relationship in variables (item 1e) at the pre-test. At the post-test, almost half of the experimental group students defined the function rule in variables. In Item 1f, students were asked to find the number of circles drawn on the 100th day. Most of the experimental and control group students could find the correct answer. In Item 1g, students were asked to construct a coordinate graph to show the relationship between the variables. None of the students could draw a correct coordinate graph. The frequency of correctness of students' answers for each sub-item in the second main problem was presented in Table 3.

Table 3.
Correctness of Students' Answers for Item 2

	Experimental Group N=20		**Control Group N=23**	
Items	**PRE**	**POST**	**PRE**	**POST**
Item 2a	25 %	50 %	21.74%	26.09%
Item 2b	10 %	45 %	13.04%	17.39%
Item 2c	30%	55%	47.83%	82.61%
Item 2d	0%	25%	0%	4.35%
Item 2e	0%	25%	0%	0%
Item 2f	35%	35%	34.78%	30.43%
Item 2g	25%	30%	43.48%	34.78%

The second main problem was about the $y=3x+2$ functional relationship. Students had difficulty in this problem to define the function rule. Both groups' pre-test scores were similar, but the experimental group showed more significant pre-to-post gains for some items. No students defined the functional relationship in words and variables correctly on the pre-test; 25% of the experimental group students could explain the function rule in words and variables on the post-test (items 2d and 2e).

In addition to the correctness, students were assigned a strategy code for their answers. Table 4 presents strategies adapted from Stephens et al. (2017). Items that students were asked to define patterns and to describe the function rule in words and variables were coded according to these codes (see Table 4).

Table 4.
Coding Scheme for Items 1 and 2

Levels	Strategy Code	Description	Example
L10	**Functional Condensed-Words (FC-W)**	Student identifies function rule in words that describe a generalized relationship between two variables.	The number of circles is two times the number of days We multiply the number of weeks by three, then add 2.
L9	**Functional Condensed – Variable (FC-V)**	Student identifies rule in variables that describe a generalized relationship between the two variables.	G×2=D, D÷2=G, G×2=Number of circles, D÷2= Number of days (H×3) + 2=P, H×3+2=P, H×3+2=amount of money, number of week×3+2=P
L8	**Functional Emergent-Words (FE-W)**	Student identifies incomplete rule in words, often describing transformation on one variable but not explicitly relating to other.	We multiply the number of days by two We multiply the number of weeks by 3, then add 2.
L7	**Functional Emergent-Variables (FE-V)**	Student identifies incomplete rules in variables, often describing transformation on one variable but not explicitly relating to other.	G×2, H×3+2
L6	**Functional Basic (FB)**	Student identifies the general relationship between variables but not the transformation between them.	Two times, half Two more than three times
L5	**Functional-Particular (FP)**	Student identifies a functional relationship using particular numbers, but there is no general statement relating variables.	2×2=4, 3×2=6,… 1×3+2=5, 2×3+2=8, 3×3+2=11….
L4	**Single Instantiation (SI)**	Student writes expressions or equation with numbers and/or unknowns that provides one instantiation of the rule but does not generally relate the two variables.	2×2=4 4×3=12 12+2=14
L3	**Covariational Relationship (CR)**	Student identifies covariation relationship. The two variables are coordinated rather than mentioned separately.	Each day the number of circles increases by 2. As the number of the day goes up by 1, the number of the circles goes up by 2. Each week the amount of money increases by 3.
L2	**Recursive Pattern General (RP-G)**	Student identifies a correct recursive pattern in either or both variables.	Increasing by twos The number of circles goes up by 2. The amount of money goes up by 3 each time.
L1	**Recursive Pattern Particular (RP-P)**	Student identifies a recursive pattern in either or both variables by referring to particular numbers only.	It goes 2, 4, 6, 8, 10 5, 8, 11, 14
	Other (O)	Student produces a strategy that differs from the above or the strategy is not discernible.	1=2,2=4,3=6,4=8 2×7=14 days 14×3=42 TL

Note. Adapted from "A Learning Progression for Elementary Students' Functional Thinking" by A. C. Stephens, N. Fonger, S. Strachota, I Isler, M. Blanton, E. Knuth, A. M. Gardiner, 2017, *Mathematical Thinking and Learning*, 19(3), p. 153.

Item 1 was about $y = 2x$ functional relationship (see Appendix A). Students were supposed to define the functional relationship between two variables (the number of days and the number of circles) and represent this relationship by table, words, variables, and graph. Item 1a asked students to determine the unknown step of the pattern. Item 1b asked students to organize a table to record data. Item 1c asked students to define patterns they saw in the table. Students were expected to explain the relationship between two variables in words in Item 1d. Students were supposed to define this relationship by using variables in Item 1e. Item 1f asked students to find the value for a further step. Students were expected to show the relationship between two variables on the coordinate graph in item 1g. In this paper, the result of items that asked to define patterns in the table, describe the relationship between variables in words, and variables will be presented. Therefore, results will be presented through functional thinking strategies (see Table 4).

Item 1c

Item 1c asked students to define patterns they saw in the table (in item 1b). Figure 2 presents the percentages of students' strategies at the pre-test and post-test.

Most experimental and control group students defined recursive patterns at pre-test and post-test. While neither the experimental group nor the control group used the functional condensed-words (FC-W) strategy at the pre-test, the frequency of the FC-W strategy was similar in the experimental group and control group at the post-test, 15%, and about 13%, respectively. However, more experimental group students used more sophisticated strategies such as functional basic (FB) and functional emergent-words (FE-W) in the post-test when asked about the patterns they saw in the table than they had at the the pre-test (10% vs. 30% at pre-test and post-test, respectively).

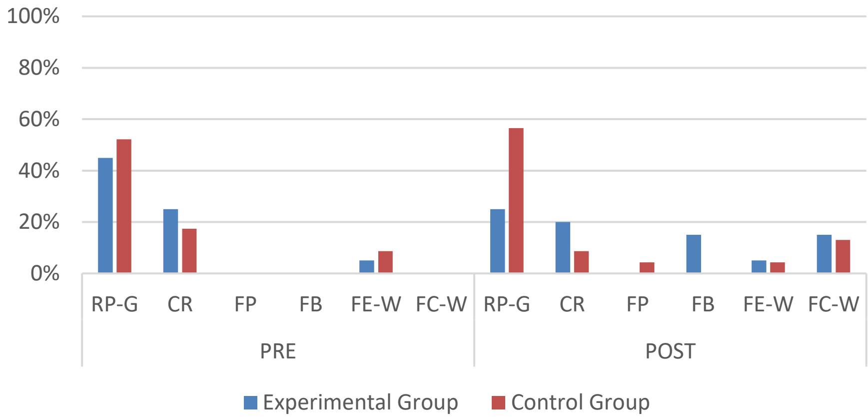

Figure 2. Percentage of Students' Strategies for Item 1c

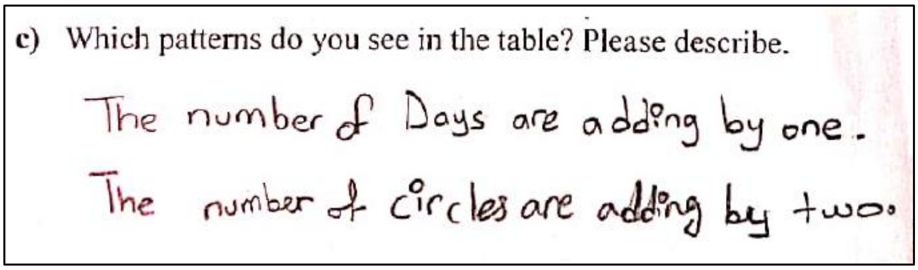

Figure 3. The Recursive Pattern-General (RP-G) response of an experimental student at the post-test

As shown in Figure 3, the student defined recursive pattern-general (RP-G) by focusing on the increase in the number of days or the number of circles separately. Moreover, some experimental group students defined the general relationship between the number of days and the number of circles but could not transform between them (see Figure 4).

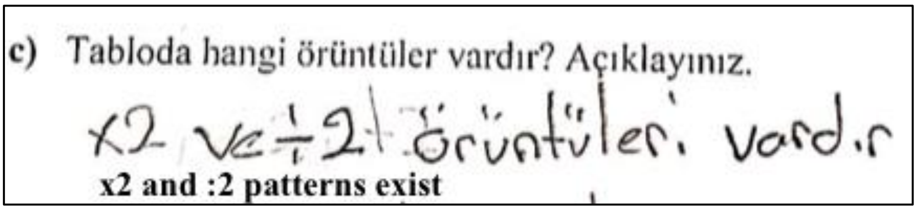

Figure 4. The Functional Basic (FB) response of an experimental student at the post-test

Item 1d

For Item 1d, students were expected to define the functional relationship between the number of days and circles in words. Figure 5 presents the students' percentages at pre-test and post-test.

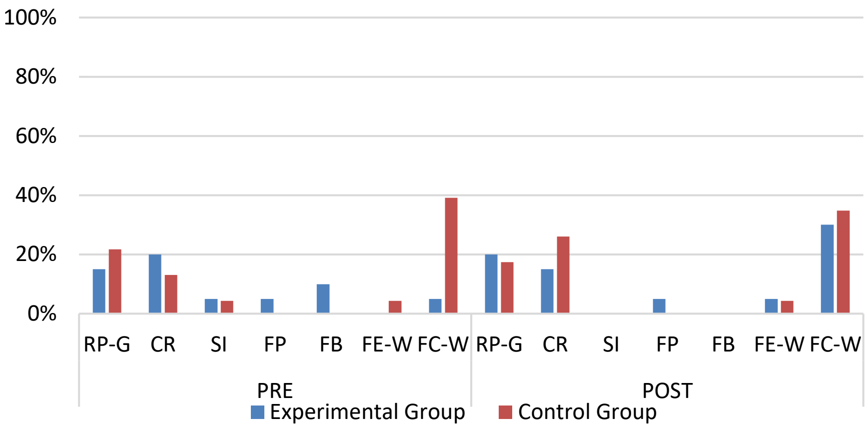

Figure 5. Percentage of Students' Strategies for Item 1d

In the control group, the percentage of the covariational relationship (CR) increased from approximately 13% at the pre-test to 26% at the post-test. However, the percentage of the covariational relationship in the experimental group decreased in the post-test. In the experimental group, the percentage of the functional condensed-words (FC-W) increased from 5% at the pre-test to 30% at the post-test, while the percentage of FC-W was about the same in the control group (39% at the pre-test, 35% at the post-test). Also, the percentage of the functional relationship strategies in total (FC-W, FE-W, FB, FP, SI) increased from 25% at the pre-test to 40% at the post-test for the experimental group. In contrast to the experimental group, the percentage of the functional relationship strategies in total (FC-W, FE-W, FB, FP, SI) decreased from about 48% at the pre-test to about 39% at the post-test for the control group. Consequently, experimental group students showed development, and they were found to use more sophisticated strategies than the control group in the post-test.

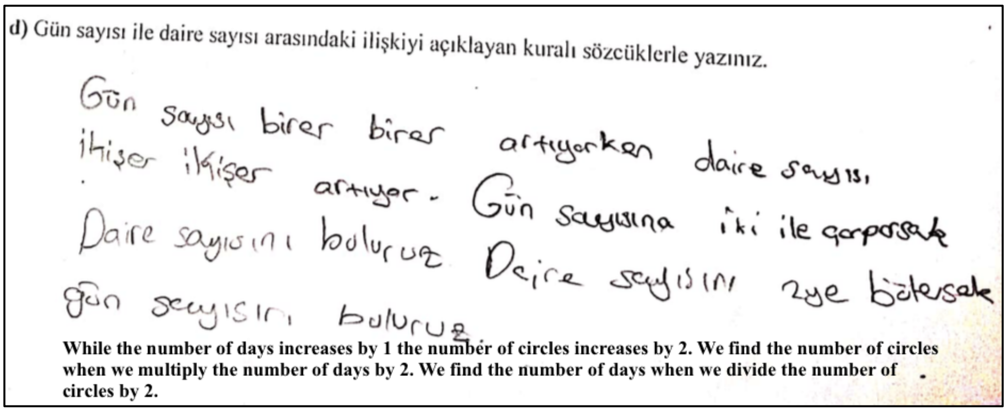

Figure 6. The Functional Condensed-Word (FC-W) strategy of an experimental group student at the post-test

As shown in Figure 6, the student defined the covariational relationship (CR) and functional condensed in words (FC-W) strategies. The student could correctly describe the relationship between the number of days and the number of circles in words.

Item 1e

Item 1e asked students to define the functional relationship between the number of days and circles by variables. Figure 7 presents the percentages of students' strategies at the pre-test and post-test.

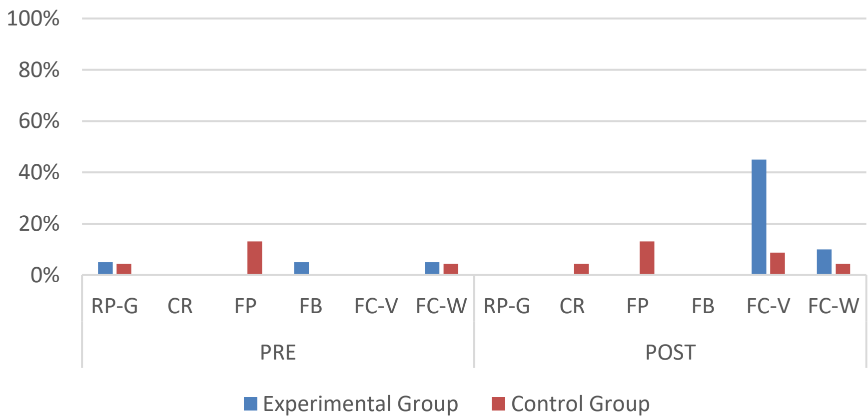

Figure 7. Percentage of Students' Strategies for Item 1e

Most of the students in both groups did not answer this part at the pre-test. None of the experimental or control students used the functional condensed-variables (FC-V) strategy at the pre-test. Approximately 9% of the control group used the FC-V strategy in the post-test, while this was 45% for the experimental group. Consequently, experimental group students were more successful than control students in defining the functional relationship using variables.

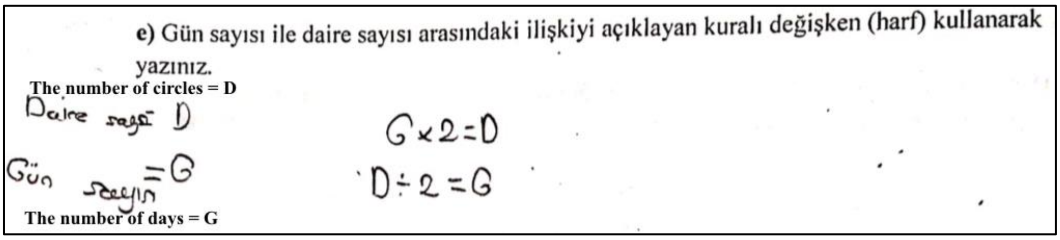

Figure 8. The Functional Condensed- Variables (FC-V) strategy of an experimental student at the post-test

As shown in Figure 8, the student used the FC-V strategy by defining two variables (G: the number of days, D: the number of circles) and generalized functional relationship in two ways using variables.

Item 2 was about the $y = 3x+2$ functional relationship (see Appendix A). Students were supposed to define the functional relationship between two variables (the number of weeks and the amount of money in the piggy bank) and represent this relationship by table, words, variables, and graph. Item 2a asked students to determine the amount of money in the piggy bank for the 2nd, 3rd, and 4th weeks. Item 2b asked students to organize a table to record data. Item 2c asked students to define patterns they saw in the table. Students were expected to explain the relationship between two variables by words in Item 2d. Students were supposed to define this relationship by using variables in Item 2e. Students were expected to find the amount of money saved at the end of week 30 by using the function rule for Item 2f. Students were expected to use the reversibility of function rule for item 2g. In this paper, similar to item 1, the focus will be on the results of items that asked to define patterns in the table describing the relationship between variables in words and variables.

Item 2c

Item 2c asked students to define patterns they saw in the table (in item 2b). Figure 9 presents the students' percentages at the pre-test and post-test.

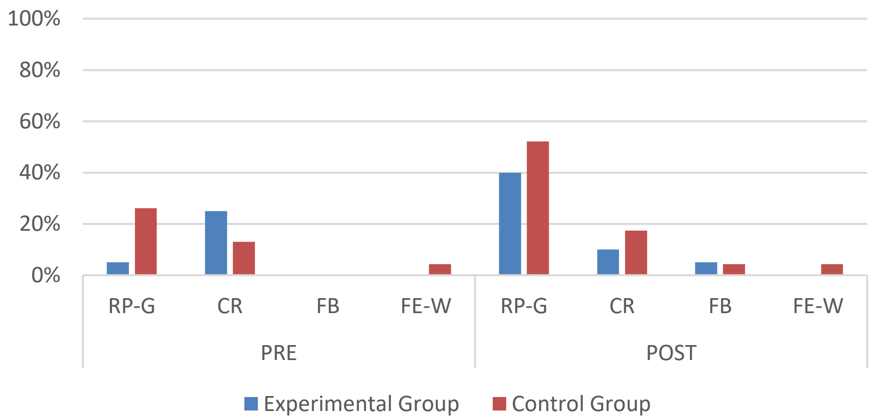

Figure 9. Percentage of Students' Strategies for Item 2c

The majority of the experimental and control group students explained the pattern by using RP-G; this increased from the pre-test to the post-test (in the experimental group from 5% to 40%, and in the control group from about 26% to about 52%). About the same ratio of students in both groups used the functional basic (FB) strategy at the post-test while none used it at the pre-test. In the post-test, the percentage of the covariational relationship (CR) strategy was higher for the control group than the experimental group (approximately 17% vs. 10%, respectively). In addition to the CR strategy, one of the control group students used the Functional Emergent- Word strategy (see Figure 10). To sum up, control group students used sophisticated strategies more frequently in item 2c (e.g., CR, FE-W).

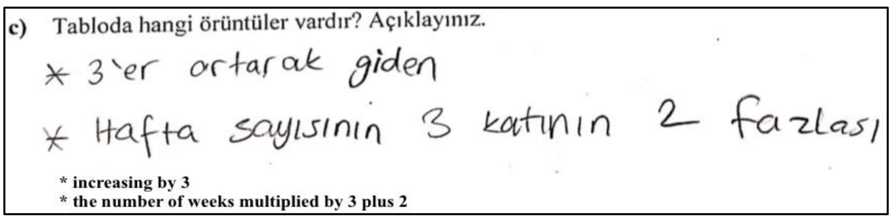

Figure 10. The Functional Emergent-Word (FE-W) strategy of a control student at the post-test

Item 2d

For Item 2d, students were expected to define the functional relationship between the number of weeks and the amount of money in the piggy bank in words.

Figure 11 presents the percentages of students' strategies at the pre-test and post-test. When asked about the function rule in words, while 25% of the experimental group used the functional condensed-words (FC-W) at the post-test, approximately 4% of the control group used it. The covariational relationship strategy decreased from 20% at the pre-test to 15% at the post-test in the experimental group. However, it doubled in the control group from about 13% at the pre-test to 26% at the post-test. None of the students used the SI strategy in the post-test, while one of the control group students did in the pre-test. FB strategy was used neither in the experimental nor in the control group at the pre-test, but one of the students from each group used this strategy at the post-test. Consequently, students had difficulty in using functional thinking strategies.

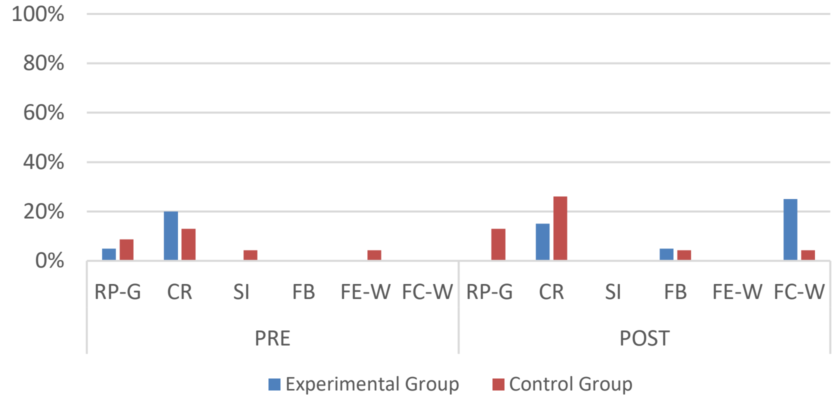

Figure 11. Percentage of Students' Strategies for Item 2d

Item 2e

For Item 2e, students were expected to define the functional relationship between the number of weeks and the amount of money in the piggy bank in variables. Figure 12 presents the percentages of students' strategies at the pre-test and post-test.

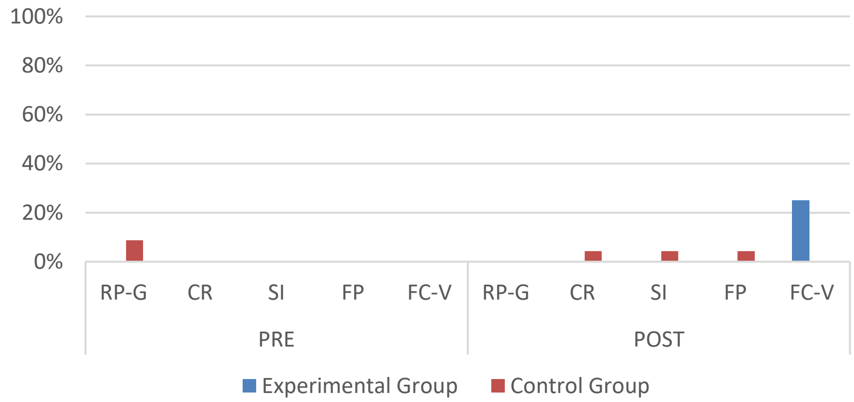

Figure 12. Percentage of Students' Strategies for Item 2e

When asked about the function rule in variables, although no control or experimental group students used the functional condensed-variable (FC-V) strategy at the pre-test, 25% of experimental group students used the FC-V strategy at the post-test. In contrast, none of the control group students used the FC-V strategy in the post-test. The experimental group students who used the FC-V strategy were the same students who wrote the correct functional relationship in words in item 2d.

In addition to the results presented above, students had difficulty defining patterns and function rules in words and variables for Item 2, so both experimental and control group students frequently used the "Other(O)" strategy. For example, in item 2c, one of the students defined the pattern as "All is increasing by 1," and another defined "7 × number of days + 2". In item 2d, students wrote an incorrect function rule, for instance, "The amount of money equals to one more than twice the number of weeks." There are also examples for students' other category answers for item 2e that asked to define the function rule in variables such as "P≠H+3" and "H+2 for once, then H+3", "H × 2 = T + 2 = P". Also, some of the students accepted the constant 2 TL as the money in the piggy bank at the end of the first week, which led them to define the function rule incorrectly.

As a result, both experimental and control group students used more sophisticated functional thinking strategies for item 1 than item 2. Although most experimental group students tended to define recursive patterns on the pre-test, they used the functional thinking strategies for items 1 and 2 on the post-test. Even though the control group students had initial success (they had a higher mean score on the pre-test) for items 1 and 2, experimental group students showed a pre-to-post gain in the test. Also, experimental group students outperformed the control group in the post-test, significantly at items 1e and 2e (using variables in defining functions rule).

4. RESULTS, DISCUSSION AND RECOMMENDATIONS

The present study aimed to investigate the effects of a functional thinking intervention on fifth-grade students' functional thinking skills. Therefore, students took a pre-test and an identical post-test. Between the tests, experimental group students received a functional thinking intervention. Although the control group's mean (M=5.65) was higher than the experimental group's mean (M=4.75) at the pre-test, the experimental group's mean (M = 7.05) was higher at the post-test than the control group (M = 6.39). In contrast to the control group, the experimental group showed a statistically significant gain between the tests. These findings support the other studies (e.g., Blanton, Stephens, et al., 2015; Stephens et al. 2017) in that the experimental group showed significant development in defining the functional relationships after the intervention.

Moreover, the descriptive results of the study revealed that the experimental group students used more sophisticated strategies in defining the functional relationships than the control group students did in the post-test. Especially in items 1e and 2e, experimental group students significantly outperformed the control group in writing the function rule in variables. In general, students were better at defining the functional relationship for $y=2x$ (item 1) than $y= 3x+2$ (item 2). This result was parallel to other studies (e.g., Blanton, Brizuela, Gardiner, Sawrey & Newman-Owens, 2015; Türkmen & Tanışlı, 2019).

Items 1c and 2c asked students to define patterns they saw in the table. Students tended to describe the recursive pattern general (L2) as "The number of circles increases by 2" for item 1c at the pre-test. This result was not a surprise because the national mathematics curriculum (MoNE, 2018) focuses on recursive patterns in 5th grade. More than half of the experimental group students could define covariational and functional relationships at the post-test. These findings were consistent with the study performed by Stephens et al. (2012), which defended that early algebra intervention helped students regard the covariational and functional relationships between two co-varying variables. On the other hand, students had difficulty describing the covariational and functional relationships between co-varying variables for item 2c. Surprisingly, control group students were more successful in defining patterns for this item in the post-test. This result can be explained by the 5th-grade curriculum's focus on recursive patterns (see MoNE, 2018, M.5.1.1.3), students' being familiar with patterns through the curriculum, and the control group students' being at a school that enrolled with GPA.

Items 1d and 2d asked students to define the function rule in words. The control group was more successful in defining the function rule in words (39% vs. 5% for the control and experimental group, respectively) at the pre-test in item 1d. The percentage of using covariational relationship (L3) in the control group increased at the post-test. However, in the experimental group, the percentage of writing the function rule (functional condensed in words [L10]) increased (from 5% to 30%) at the post-test. This result was similar to the "main path," that is, students tended to define the recursive relationship at the beginning. Then, they shifted toward correspondence thinking (Stephens et al., 2017). Similarly, in item 2d, while the control group showed an increase (13% vs. 26% for pre-test and post-test, respectively) in the covariational relationship (L3), the experimental group showed an increase (0% vs. 25% for pre-test and post-test, respectively) in writing the function rule (functional condensed in words [L10]).

Items 1e and 2e asked students to write the function rule in variables. As expected, no students defined the function rule in variables at the pre-test. For Item 1e, experimental students showed significant (from 0% to 45%) development between tests in writing the function rule in variables. Surprisingly, two students from the control group could write function rules in variables at the post-test in item 1e. All students had difficulty defining the function rule in variables for Item 2e. Stephens et al. (2017) found that students were more successful writing the function rule in variables than words. Although this finding was found parallel for item 1e, in the current study, for item 2e, for the experimental group, the percentage of defining the function rule in variables (25%) was found equal to defining the function rule in words (25%) at the post-test. Specifically, for item 2e, the same students who defined the function rule in words could write it in variables.

Studies (e.g., Çelik & Güneş, 2013; Dede & Argün, 2003) mainly investigated students' difficulties in algebra and algebraic concepts. However, this study showed that students could use variables and equations to generalize functional relationships after a functional thinking intervention. The present study found that the functional thinking intervention helped fifth-grade students to gain an algebraic approach to functional relationships. Functional thinking promotes the generalization of functional relationships by multiple representations (words, variables, tables, graphs, and drawings). In conclusion, functional thinking could help students develop algebraic thinking if introduced early in the curriculum, especially through contextual problems like those that were used in this study. Therefore, curriculum developers could consider the results of this study in that regard. Lastly, the researchers developed and implemented the functional thinking intervention in this study. This study revealed that fifth-grade students could develop functional thinking through an appropriate intervention. Thus, teachers' readiness is important to provide such learning opportunities to their students. Future studies should focus on teacher education and teachers' designing and implementing such activities. Teachers are key in helping students develop algebraic thinking (Blanton

& Kaput, 2005). Teacher educators should pay attention to this matter, and professional development programs should be designed to focus on helping teachers foster algebraic and, more specifically, functional thinking in classrooms.

Research and Publication Ethics Statement

This study was carried out within the scope of the Master thesis of the first author and the necessary official permissions were obtained from the relevant institutions. All authors declare that they adhered to the principles of research and publication ethics throughout the research.

Contribution Rates of Authors to the Article

This study reported here was based on the first author's master's study under the supervision of the second author.

Statement of Interest

The authors declare that they have no competing interests.

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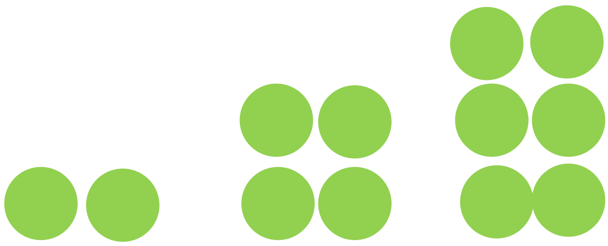
APPENDIX A. FUNCTIONAL THINKING TEST

AD-SOYAD:

1) **Selin, her gün okulda dairelerden oluşan bir resim çiziyor. Selin'in ilk üç günde çizdiği resimler aşağıdaki gibidir:**



1.Gün 2. Gün 3. Gün

a) Selin'in, 5. günde çizeceği resimdeki daire sayısını bulunuz.

b) Elde ettiğiniz verileri yandaki tabloya yazınız.

Gün sayısı	Daire sayısı

c) Tabloda hangi örüntüler vardır? Açıklayınız.

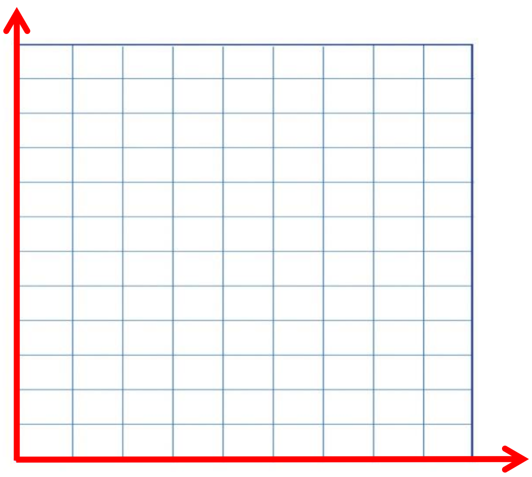
d) Gün sayısı ile daire sayısı arasındaki ilişkiyi açıklayan kuralı sözcüklerle açıklayınız.

e) Gün sayısı ile daire sayısı arasındaki ilişkiyi açıklayan kuralı değişken kullanarak yazınız.

f) Selin'in, okulun 100. gününde çizeceği resimde kaç tane daire olmalıdır?

g) Gün sayısı ile daire sayısı arasındaki ilişkiyi grafikle gösteriniz.

Daire sayısı



Gün sayısı

Note. Adapted from "A Learning Progression for Elementary Students' Functional Thinking" by A. C. Stephens, N. Fonger, S. Strachota, I. Isler, M. L. Blanton, E. Knuth, and A. M. Gardiner, 2017, *Mathematical Thinking and Learning*, 19(3), p.149.

2) **Mert'in en başta kumbarasında 2 TL'si vardır. Mert'in babası ev işlerinde yardımcı olduğu için her hafta Mert'e 3 TL verme kararı almıştır ve Mert aldığı harçlıkları kumbarasında biriktirerek toplam parası ile bir bisiklet almaya karar veriyor. Buna göre;**

a) Mert'in; 2 hafta, 3 hafta ve 4 hafta sonunda kumbarasındaki toplam para miktarı ne kadardır?

b) Elde ettiğiniz bilgileri tablo oluşturarak düzenleyiniz.

c) Tabloda hangi örüntüler vardır? Açıklayınız.

d) Hafta sayısı ile Mert'in kumbarasındaki toplam para miktarı arasındaki ilişkiyi sözcüklerle açıklayınız.

e) Hafta sayısı ile Mert'in kumbarasındaki toplam para miktarı arasındaki ilişkiyi değişken kullanarak yazınız.

f) Mert, 30 hafta sonunda kumbarasındaki toplam para miktarı ne kadar olur?

g) Mert'in almak istediği bisikletin fiyatı 95 TL ise Mert kaç haftanın sonunda bu bisikleti satın alabilir?

APPENDIX B. SAMPLE LESSON PLAN

DERS PLANI 5:

Bu dersin kazanımları;

* Elde ettiği verileri tablo kullanarak düzenleyebilme
* Değişkenleri ve değişken rollerini sözel olarak tanımlayabilme
* Yinelemeli örüntüleri (recursive pattern) sözel olarak tanımlayabilme
* Kovaryasyonel (birlikte değişim) ilişkileri (covariational relationship) sözel olarak tanımlayabilme
* Fonksiyon kuralını sözel ve sembolik olarak tanımlayabilme ($y=2x+1$ ilişkisini kurabilme, sözel ve sembolik olarak tanımlayabilme)
* Bağımlı değişkene ait bir değer verildiğinde bağımsız değişkene ait değeri hesaplayabilmedir.

Öğrencilerden beklenen ön bilgiler:

* Öğrenciler elde ettiği verileri tabloya yerleştirebilir
* Tablodaki örüntüyü ifade edebilir.
* Değişken içeren ifadeler yazabilir.
* Fonksiyon kuralını sözel ve sembolik olarak tanımlayabilme ($y=x$, $y=2x$, $y=x+1$ ilişkisini kurabilme, sözel ve sembolik olarak tanımlayabilme)

Öğretim Tekniği: Grup çalışması, Keşfetme, Tartışma
Materyal: Etkinlik kâğıtları, değerlendirme kartları, kurdele, makas
Süre: 80 dk.

Başlangıç (5dk.):

- Önceki derste kurdele kesimi süreci, bu sürecin sonunda elde edilen sonuçlar ve ders sonundaki değerlendirme kartlarından yola çıkılarak tekrar yapılır.
- Kavram yanılgısı olan noktalar varsa kısa sürede geri dönüşler yapılarak yeni ders başlangıcı yapılır.

Gelişme (65 dk.):

- Önceki derste yapılan kurdele kesme etkinliği hatırlatılarak bu sefer düğümlü kurdeleler kullanarak kesme işlemi yaparsak nasıl bir ilişki ortaya çıkar şeklinde bir giriş yapılabilir.
- Öğrenciler grup çalışması yapmak için 4 kişilik gruplara ayrılır.
- Her gruba 4 farklı renkte (mavi, kırmızı, pembe, sarı) kurdeleler ve birer makas verilir.
- Çalışmaları sırasında kullanacakları etkinlik kâğıtları dağıtılır.
- Her kurdelede 1 düğümün olmasına dikkat çekilir.
- Hiç kesim yapmadan kaç parça olduğu sorulur.
- İlk kesim tüm sınıfla birlikte yapılır.
- Her gruptan kırmızı kurdeleyi almaları ve düğüm yerinden ikiye katlamaları istenir. Daha sonra düğüm dışındaki bir noktadan 1 kere kesim yapılması istenir ve kaç parça kurdele elde edildiği üzerine konuşulur. Her grubun 3 parça elde ettiğinden emin olunmalıdır.
- Bulunan değerler tabloya yazılır.
- Mavi kurdele için aynı şekilde katlanarak 2 kesim, pembe kurdele için 3 kesim ve sarı kurdele için 4 kesim yapmaları gerektiği anlatılır.
- Öğrencilerin buldukları sonuçları tabloya yazmaları beklenmektedir.

- Grup çalışması sırasında gruplar gözlemlenir.
- Daha sonra tablodaki veriler arasında bir örüntü olup olmadığı var ise tanımlamaları istenir.
- Örüntüyü sadece parça sayısına odaklanarak yani tabloda yukarıdan aşağıya ilerleyen yinelemeli bir örüntü olarak tanımlayabilirler.
- Kesim sayısı ve parça sayısı arasında ilişki olup olmadığı sorulur.
- Kesim sayısı ve parça sayısı arasındaki ilişkiye dikkat çekmek için *"Kesim sayısı .... artarken elde edilen parça sayısı .... şeklinde artar."* İfadesi kullanılabilir.
- Herhangi sayıda yapılan kesimden elde dilen parça sayısını nasıl ifade edebilecekleri ya da n tane kesim sonucunda kaç parça elde edilir şeklinde sorular yöneltilerek aradaki ilişkiyi sembolik olarak ifade etmeleri konusunda yardımcı olunabilir.
- Parça sayısı (P), Kesim sayısı (K) ile gösterilebilir. İlişkiyi açıklayan sembolik gösterim "*P=2xK+1*" ya da "*y=2x+1*" olarak tanımlanabilir.
- Kuralı bulduktan sonra 20 kesim sonucunda kaç tane parça elde edileceği sorusunun cevabını kuralı kullanarak vermeleri beklenir.

Bitiş (10 dk.):

- Ders içindeki etkinlik süreci ve ulaşılan fonksiyonda "*y=2x+1*" değişkenlerin anlamı, değişkenler arasındaki ilişki tekrar edilerek çıkış kartları dağıtılır.
- Çıkış kartların çözümü için zaman verilir.

ETKİNLİK KÂĞIDI 5

a) 1 Kesim yapıldığında kaç parça elde edilir?

- 2 Kesim yapıldığında kaç parça elde edilir?
- 3 Kesim yapıldığında kaç parça elde edilir?
- 4 Kesim yapıldığında kaç parça elde edilir?

b) Elde ettiğiniz verileri tablo oluşturarak düzenleyiniz.

c) Tablodaki veriler arasında bir örüntü var mı? Var ise bu örüntüyü tanımlayınız?

d) Kesim sayısı ve parça sayısı arasında ilişki var mıdır? Var ise bu ilişkiyi nasıl tanımlarsınız?

e) Bu ilişkiyi <u>**değişken kullanarak**</u> nasıl ifade edebilirsiniz?

f) 20 defa kesim yapıldığında kaç parça elde edilir?

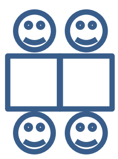
Note. Reprinted from "The string task: Not just for high school" by Isler, I., Marum, T., Stephens, A., Blanton, M., Knuth, E., & Gardiner, A. M., 2015, *Teaching Children Mathematics*, *21*(5), 285.

ÇIKIŞ KARTI 5

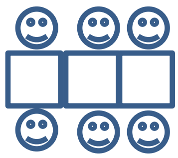
Nehir arkadaşlarını davet ettiği bir doğum günü partisi planlamaktadır. Partiden önce herkes için yeterli sayıda oturma yeri olup olmadığından emin olmak istiyor. Nehir kare şeklindeki masalara sahiptir.

Bir masada şekildeki gibi 2 kişi oturmaktadır.

Nehir bir masa daha eklediğinde; 2 masada şekildeki gibi 4 kişi oturmaktadır.



Nehir ikinci masaya bir masa daha eklerse 3 masada şekildeki gibi 6 kişi oturmaktadır.



a) Aşağıdaki tabloyu doldurarak farklı sayıdaki masalara oturabilecek kişi sayısını gösteriniz.

Masa Sayısı	Kişi sayısı
1	2
2	4
3	
4	
5	
6	
7	

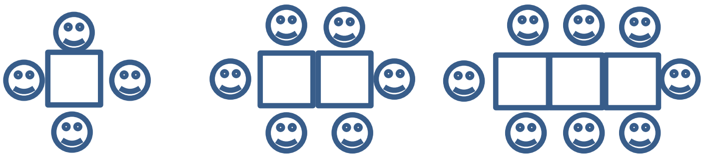
b) Oluşturduğunuz tabloda bir örüntü var mı? Var ise bu örüntüyü tanımlayınız.

c) Masa sayısı ile kişi sayısı arasında bir ilişki var mıdır? Var ise bu ilişkiyi sözcüklerle nasıl tanımlarsınız?

d) Bu ilişkiyi değişken kullanarak nasıl ifade edebilirsiniz? Bu değişkenler neyi ifade ediyor?

e) Bu parti için 100 masa birleşik olarak (yukarıdaki gibi) dizilirse kaç kişi partiye katılabilir?

Nehir masanın uçlarına iki kişinin daha oturması durumunda daha fazla kişiyi davet edebileceğini fark etmiştir. Örneğin, eğer Nehir şekildeki gibi 2 masayı birleştirirse 6 kişi oturabiliyor; 3 masayı birleştirirse 8 kişi oturabiliyor.



f) Yeni durum c ve d şıklarında yazdığın kuralı nasıl etkiler?

g) Yeni durumda masa sayısı ve kişi sayısı arasındaki ilişkiyi sözcüklerle nasıl tanımlarsınız?

h) Yeni durumda masa sayısı ve kişi sayısı arasındaki ilişkiyi değişken kullanarak nasıl ifade edebilirsiniz? Bu değişkenler neyi ifade ediyor?

i) Yeni durumda bu parti için 100 masa birleşik olarak (yukarıdaki gibi) dizilirse kaç kişi partiye katılabilir?

j) Yeni durumda bu partiye 100 kişinin katılabilmesi için kaç tane masa gereklidir?

Note. Adapted from “The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in third grade” by M. Blanton, A. Stephens, E. Knuth, A. M. Gardiner, I. Isler, and J. S. Kim, 2015, *Journal for research in Mathematics Education*, *46*(1), pp. 85-86.